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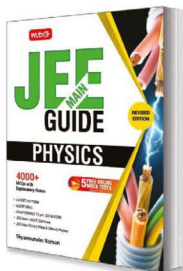
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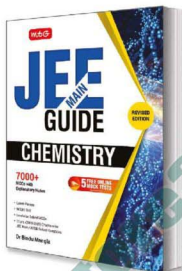
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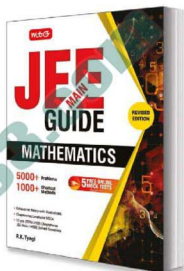
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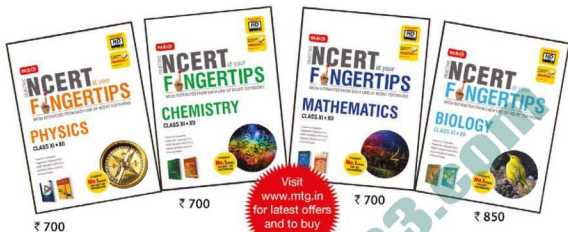
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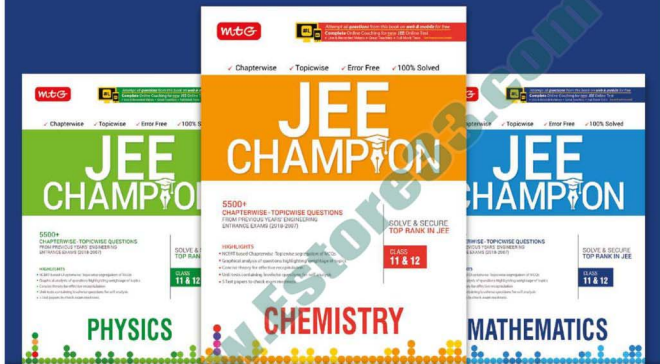


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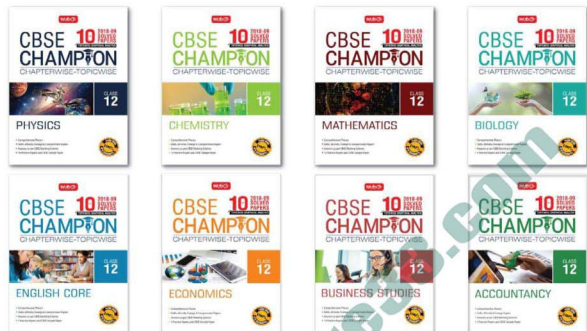


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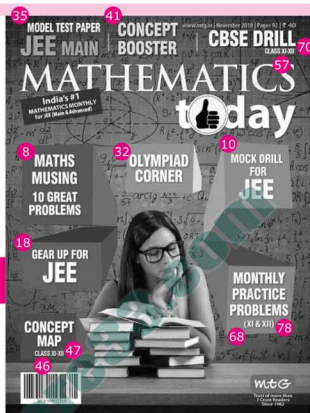
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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 191

JEE MAIN

1. If two distinct chord from the point $(4, 4)$ on the parabola $y^2 = 4x$ are bisected by the line $y = mx$, then the set of values of m is given by

- (a) $(0, \infty)$ (b) R
(c) $(-2, 2)$ (d) $\left(\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right)$

2. One hundred identical coins, each with probability, p of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

- (a) $\frac{1}{2}$ (b) $\frac{49}{101}$ (c) $\frac{50}{101}$ (d) $\frac{51}{101}$

3. A differentiable function satisfies $3f^2(x)f'(x) = 2x$. Given $f(2) = 1$, then the value of $f(3)$ is

- (a) $\sqrt[3]{24}$ (b) $\sqrt[3]{6}$ (c) 6 (d) 2

4. The range of the function

$$f(x) = \cos x \left\{ \sin x + \sqrt{(\sin^2 x + \sin^2 \alpha)} \right\} \text{ is}$$

- (a) $[-\sqrt{1+\sin^2 \alpha}, \sqrt{1+\sin^2 \alpha}]$
(b) $[-\sqrt{1+\sin^2 \alpha}, 0]$
(c) $[0, \sqrt{1+\sin^2 \alpha}]$ (d) None of these

5. Through the point $P(h, k, l)$ a plane is drawn at right angles to OP to meet co-ordinate axes at A, B and C . If $OP = p$, then the area of the ΔABC is

- (a) $\frac{p^5}{2hkl}$ (b) $\frac{p^5}{hkl}$ (c) $\frac{p^3}{2hkl}$ (d) $\frac{p^3}{hkl}$

JEE ADVANCED

6. If $x^2 + 2hxy + y^2 = 0$ represents the equations of the straight lines through the origin which make an angle α with the straight line $y + x = 0$, then

- (a) $\sec 2\alpha = h$ (b) $\cos \alpha = \sqrt{\frac{1+h}{2h}}$
(c) $2\sin \alpha = \sqrt{\frac{1+h}{h}}$ (d) $\cot \alpha = \sqrt{\frac{h+1}{h-1}}$

COMPREHENSION

If $\lim_{x \rightarrow 0} f(x) = 2$ where $f(x) = \frac{axe^x - b \log(1+x) + cxe^{-x}}{x^2 \sin x}$ and

a, b, c are real numbers.

7. $\lim_{x \rightarrow 0} x \sin f(x) =$
(a) 0 (b) 1 (c) 3 (d) 4

8. Which of the following is true?

- (a) $b = a + c$ (b) $c = a^2$
(c) $c = 3a$ (d) All of these

INTEGER TYPE

9. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then $|z_1 + z_2 + z_3|$ is equal to

MATRIX MATCH

10. α, β are the maximum and minimum values of

$$f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1 + \cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1 + \sin 2x \end{vmatrix},$$

then match the following:

	Column-I	Column-II
P.	$\alpha + \beta^{87}$	1. 6
Q.	$\alpha^2 - 3\beta^{11}$	2. 2
R.	$f'\left(\frac{\pi}{2}\right)$	3. 4
S.	$f\left(\frac{\pi}{2}\right)$	4. -2

- P Q R S P Q R S
(a) 3 1 4 2 (b) 1 3 2 4
(c) 2 1 3 4 (d) 3 4 1 2

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Myth

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Suggestion - Do not study for marks, try to enjoy the subjects.

Reality

Please visit youtube to watch more myth and reality discussions on KCS EDUCATE channel.

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Knowledge Quiz - 12

Find all pairs $x, y \in \mathbb{Z}$, so that

$$x^3 + x^2y + xy^2 + y^3 = 8(x^2 + xy + y^2 + 1).$$

Please send your detailed solution before 15th November 2018 to quiz@kcseducate.in along with your name, father's name, class, school, address and contact number.

Winner Knowledge Quiz - 11

- Akanksha Das, (Class - VIII), Auxilium Convent Bandel, Hooghly, West Bengal.
- Riddhi Roy, (Class - X), Michael nagar Sikshaniketan English Medium High School, Kolkata, West Bengal.
- Chirag Arora, (Class - XI), Delhi Public School Ranipur, Haridwar, Uttarakhand.

Shalini Agrawal brings laurels

■ Business Reporter
Bhilai, Oct 7

Bhilai: A classmate and a student of KCS Educate, Shalini Agrawal brought laurels to the city by participating in the AIM-SIRIUS Innovation Festival-2018 during Russian President Vladimir Putin visit to India. Shalini a student of class-XI during the festival prepared projects along with other Indian and Russian students and presented it before the Russian President Putin and Prime Minister of India Narendra Modi.

In the above festival 10 Indian students were selected on the basis of their performance in educational and other activities. Out of the 10 students 5 were girls of which one was Shalini. This programme was organised under ITEP (India's Total Innovation Mission (AIM) in association with Russian students from SIRIUS Education Center. The AIM-SIRIUS Innovation Festival-2018 was organised at New Delhi from October 1 to 5.

Shalini Agrawal is daughter of renowned pediatrician of city, Dr. Rajkumar Agrawal and Dr. Snehalata Agrawal. She was topper in Class-10th and wants to pursue her engineering from an IIT for which she is taking coaching from KCS Educate. Director of KCS, Arunish Saxena has congratulated Shalini and bestowed his best wishes for a bright future.

Thanks for Making us Proud !!!



पुतिन ने की शालिनी के प्रोजेक्ट की सराहना

देश के सम्प्रति वरत विद्यार्थियों के साथ उनके 10 कक्षा समकक्षी ने अड्डाइट्टी दिल्ली में अन्तराष्ट्रीय संघी परियोजना कार्य पर अपनी विशिष्ट सोच एवं सोशल को प्रस्तुत किया। यह प्रोजेक्ट भी तैयार किया गिरे इन्वेस्टमन्ट फेस्टिवल-2018 में प्रस्तुत होना है। शालिनी के प्रोजेक्ट को रूस के राष्ट्रपति पुतिन ने सराहा। यह होना-रार डॉ. स्नेहालता और राजकुमार अग्रवाल की सुरुष है। शालिनी महार के नयी कोषिका संस्थान केसीएस एजुकेटर्स की छात्रा है।

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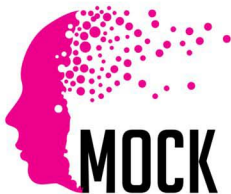
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JEE MAIN 2019

Between
6th to 20th JanuaryBetween
6th to 20th April

MOCK DRILL FOR JEE

* ALOK KUMAR

1. The values of 'a' for which the roots of the equation $(a+1)x^2 - 3ax + 4a = 0$ ($a \neq -1$) are real and greater than 1 is

- (a) $\left[-\frac{10}{7}, 1\right]$ (b) $\left[-\frac{12}{7}, 0\right]$
(c) $\left[-\frac{16}{7}, -1\right)$ (d) $\left[-\frac{16}{7}, 0\right)$

2. If $f(x)$ is a polynomial of least degree such that

$$f(r) = \frac{1}{r}, r = 1, 2, 3, \dots, 9, \text{ then } f(10) =$$

- (a) 1 (b) 1/2 (c) 1/10 (d) 1/5

3. If the equations $x^2 + 2\lambda x + \lambda^2 + 1 = 0$, $\lambda \in R$ and $ax^2 + bx + c = 0$ where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is

- (a) (0, 2) (b) $(\sqrt{3}, 3)$
(c) $(2\sqrt{2}, 3\sqrt{2})$ (d) $(0, \infty)$

4. If $a_k = \frac{1}{k(k+1)}$, for $k = 1, 2, 3, \dots, n$, then $\left(\sum_{k=1}^n a_k\right)^2 =$

- (a) $\frac{n}{n+1}$ (b) $\frac{n^2}{(n+1)^2}$
(c) $\frac{n^4}{(n+1)^4}$ (d) $\frac{n^6}{(n+1)^6}$

5. The ratio of sum of first three terms of a G.P. to the sum of first six terms is 64 : 91, the common ratio of G.P. is

- (a) 1/4 (b) 3/4 (c) 5/4 (d) 7/4

6. The number of terms common between the two series $2 + 5 + 8 + \dots$ up to 50 terms and the series $3 + 5 + 7 + 9 + \dots$ up to 60 terms is

- (a) 24 (b) 26
(c) 25 (d) none of these

7. If $p + q + r = 0$ and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$,

then $k =$

- (a) 0 (b) abc
(c) pqr (d) $a + b + c$

8. If A is a skew-symmetric matrix of order 3, then the matrix A^4 is

- (a) skew symmetric (b) symmetric
(c) diagonal (d) none of those

9. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are two matrices

such that $AB = BA$ and $c \neq 0$, then value of $\frac{a-d}{3b-c}$ is

- (a) 0 (b) 2 (c) -2 (d) -1

10. The angle of elevation of a cloud from a point h m above the surface of a lake is θ and the angle of depression of its reflection in the lake is ϕ . The height of the cloud is

- (a) $\frac{h \sin(\phi + \theta)}{\sin(\phi - \theta)}$ (b) $\frac{h \sin(\phi - \theta)}{\sin(\phi + \theta)}$
(c) $\frac{h \sin(\theta + \phi)}{\sin(\theta - \phi)}$ (d) $\frac{h \sin(\theta - \phi)}{\sin(\theta + \phi)}$

* Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

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11. If $x_1, x_2, x_3, \dots, x_n$ are in A.P. whose common difference is α , then the value of

$$\sin \alpha [\sec x_1 \sec x_2 + \sec x_2 \sec x_3 + \dots + \sec x_{n-1} \sec x_n] =$$

- (a) $\frac{\sin n\alpha}{\cos x_n \cos x_1}$ (b) $\frac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$
(c) $\frac{\sin(n+1)\alpha}{\cos x_n \cos x_1}$ (d) $\frac{\cos(n-1)\alpha}{\cos x_n \cos x_1}$

12. If $\angle C = 90^\circ$ in $\triangle ABC$, then

$$\tan^{-1} \left(\frac{a}{b+c} \right) + \tan^{-1} \left(\frac{b}{c+a} \right) \text{ is equal to}$$

- (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) π

13. If $\sin 2\beta$ is the G.M. between $\sin \alpha$ and $\cos \alpha$, then $\cos 4\beta$ is equal to

- (a) $2 \sin^2 \left(\frac{\pi}{4} - \alpha \right)$ (b) $2 \cos^2 \left(\frac{\pi}{4} - \alpha \right)$
(c) $2 \cos^2 \left(\frac{\pi}{2} + \alpha \right)$ (d) $2 \sin^2 \left(\frac{\pi}{4} + \alpha \right)$

14. If $\cos^6 \alpha + \sin^6 \alpha + k \sin^2 2\alpha = 1 \forall \alpha \in (0, \pi/2)$, then k is

- (a) $3/4$ (b) $1/4$ (c) $1/3$ (d) $1/8$

15. If $f(x) = \frac{\sin 3x}{\sin x}$, $x \neq n\pi$, then the range of $f(x)$ for real values of x is

- (a) $[-1, 3]$ (b) $(-\infty, -1)$
(c) $(3, \infty)$ (d) $[-1, 3]$

16. A line passes through $(2, 0)$. The slope of the line, for which its intercept between $y = x - 1$ and $y = -x + 1$ subtends a right angle at the origin, is/are

- (a) $\sqrt{3}$ (b) $-2\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{2}{\sqrt{3}}$

17. The range of $\frac{x^2 - x + 1}{x^2 + x + 1}$ is

- (a) $\left[\frac{1}{3}, 3 \right]$ (b) $\left[\frac{1}{3}, 1 \right]$
(c) $[1, 3]$ (d) $\left(-\infty, \frac{1}{3} \right] \cup [3, \infty)$

18. The domain of the function

$$f(x) = \sqrt{10 - \sqrt{x^4 - 21x^2}} \text{ is}$$

- (a) $[5, \infty)$
(b) $[-\sqrt{21}, \sqrt{21}]$
(c) $[-5, -\sqrt{21}] \cup [\sqrt{21}, 5] \cup \{0\}$
(d) $(-\infty, -5]$

19. $\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$ is equal to ($a > 0$)

- (a) $\log_e a$ (b) 1 (c) 0 (d) ∞

20. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to

- (a) $1/6$ (b) $1/2$ (c) 2 (d) $-1/2$

21. Let $|a_1 \sin x + a_2 \sin 2x + \dots + a_8 \sin 8x| \leq |\sin x|$ for $x \in R$. Define $p = a_1 + 2a_2 + 3a_3 + \dots + 8a_8$, then p satisfies

- (a) $|p| \leq 1$ (b) $|p| < 1$ (c) $|p| > 1$ (d) $|p| \geq 1$

22. If $f(x)$ be positive, continuous and differentiable on the interval (a, b) . If $\lim_{x \rightarrow a^+} f(x) = 1$ and $\lim_{x \rightarrow b^-} f(x) = 3^{1/4}$,

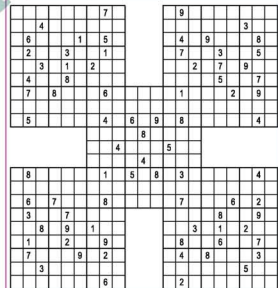
also $f'(x) > (f(x))^3 + \frac{1}{f(x)}$, then

SAMURAI SUDOKU



Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9×9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3×3 box should contain one of each digit.

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(a) $b - a > \frac{\pi}{24}$

(b) $b - a < \frac{\pi}{24}$

(c) $b - a = \frac{\pi}{12}$

(d) $b - a = \frac{\pi}{24}$

23. The coordinates of the point on the parabola $y = x^2 + 7x + 2$, which is nearest to the straight line $y = 3x - 3$ are

(a) $(-2, -8)$

(b) $(1, 10)$

(c) $(2, 20)$

(d) $(-1, -4)$

24. The normals at two points P and Q of a parabola $y^2 = 4ax$ meet at (x_1, y_1) on the parabola. Then $PQ^2 =$

(a) $(x_1 + 4a)(x_1 + 8a)$

(b) $(x_1 + 4a)(x_1 - 8a)$

(c) $(x_1 - 4a)(x_1 + 8a)$

(d) $(x_1 - 4a)(x_1 - 8a)$

25. The complex number $a + i, a - i, 1 + ai, 1 - ai$ where $a \in \mathbb{R}$ taken in that order on the argand plane represent the vertices of a parallelogram if

(a) $a = 1$

(b) $a = -1$

(c) $a = 0$

(d) none of these

26. If the complex number z satisfies $z + |z| = 2 + 8i$, then the value of $|z|$ is

(a) 8

(b) 17

(c) 15

(d) 24

27. The value of $^{12}C_2 + ^{13}C_3 + ^{14}C_4 + \dots + ^{999}C_{999}$ is

(a) $^{1000}C_{11} - 12$

(b) $^{1000}C_{11} + 12$

(c) $^{999}C_{11} - 12$

(d) $^{1000}C_{999}$

28. The greatest integer less than or equal to $(\sqrt{2} + 1)^6$ is

(a) 194

(b) 195

(c) 196

(d) 197

$$29. \sum_{m=1}^n \left(\sum_{k=1}^m \left(\sum_{p=k}^m {}^nC_m \cdot {}^mC_p \cdot {}^pC_k \right) \right) =$$

(a) $3^n - 2^n$

(b) $4^n - 3^n$

(c) $3^n + 2^n$

(d) $4^n - 1$

30. Two numbers a and b are chosen from the set of $\{1, 2, 3, \dots, 3n\}$. In how many ways can these integers be selected such that $a^2 - b^2$ is divisible by 3?

(a) $\frac{3}{2}n(n+1) + n^2$

(b) $\frac{3}{2}n(n-1) + n^2$

(c) $\frac{1}{2}n(n+1) - n^2$

(d) $\frac{1}{2}n(n-1) + n^2$

SOLUTIONS

1. (c) : $D = 9a^2 - 16a(a+1) \geq 0$,

$$\Rightarrow a(7a+16) \leq 0$$

...(i)

Also, $x_1 > 1, x_2 > 1$, where $x_1 + x_2 = \frac{3a}{a+1}, x_1 x_2 = \frac{4a}{a+1}$

$$\Rightarrow x_1 + x_2 - 1 > 0 \text{ and } (x_1 - 1)(x_2 - 1) > 0$$

$$\Rightarrow \frac{2a-1}{a+1} > 0 \quad \dots(\text{ii})$$

$$\text{and } \frac{2a+1}{a+1} > 0 \quad \dots(\text{iii})$$

Solving (i), (ii) and (iii), we get

$$-\frac{16}{7} \leq a < -1$$

2. (d) : Here $xf(x) - 1 = 0$ has roots 1, 2, 3, ..., 9

$$\therefore xf(x) - 1 = A(x-1)(x-2) \dots (x-9) \quad \dots(\text{i})$$

$$\text{Put } x = 0 \text{ in (i)} \Rightarrow A = \frac{1}{9!}$$

$$\text{Put } x = 10 \text{ in (i)} \Rightarrow 10f(10) - 1 = 1 \Rightarrow f(10) = \frac{1}{5}$$

3. (a) : $(x + \lambda)^2 + 1 = 0$ has clearly imaginary roots. So, both roots of the equations are common

$$\therefore \frac{a}{1} = \frac{b}{2\lambda} = \frac{c}{\lambda^2 + 1} = k \text{ (say)}$$

$$\text{Then } a = k, b = 2\lambda k, c = (\lambda^2 + 1)k$$

$\therefore a, b, c$ are lengths of sides of triangle [Given]

$$\therefore a + b > c \Rightarrow 2\lambda + 1 > \lambda^2 + 1 \Rightarrow \lambda^2 - 2\lambda < 0 \Rightarrow \lambda \in (0, 2)$$

The other conditions will also imply same relation.

$$4. (b) : \left(\sum_{k=1}^n a_k \right)^2 = \left[\sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) \right]^2 = \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} \right)^2 = \frac{n^2}{(n+1)^2}$$

$$5. (b) : \text{Given, } \frac{S_3}{S_6} = \frac{64}{91} = \frac{a(r^3 - 1)}{a(r^6 - 1)}$$

$$\Rightarrow \frac{(r^3 - 1)}{(r^3 + 1)(r^3 - 1)} = \frac{64}{91} \Rightarrow r^3 = \frac{27}{64} \therefore r = \frac{3}{4}$$

6. (d) : Let series 1 : 2, 5, 8, ... upto 50 terms and series 2 : 3, 5, 7, ... upto 60 terms

$\therefore$ Common series in series 1 and series 2 is

$$5, 11, 17, \dots, 119$$

Now, 40th term of series 1 = 59th term of series 2

= 119 = last term of common series

$$\therefore a_n = 5 + (n-1)d \Rightarrow 119 + 1 = 6n \Rightarrow n = 20$$

$\therefore$ Number of common terms is 20.

$$7. (c) : \text{Given } p + q + r = 0 \Rightarrow p^3 + q^3 + r^3 = 3pqr$$

$$\text{Consider } \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3 + b^3 + c^3 - 3abc)$$

$$= pqr \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \Rightarrow k = pqr$$

8. (b): We have $A^T = -A$

$$\therefore (A^4)^T = (A \cdot A \cdot A \cdot A)^T = A^T A^T A^T A^T \\ = (-A)(-A)(-A)(-A) = (-1)^4 A^4 = A^4$$

9. (d): Consider $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

Given $AB = BA \Rightarrow a + 2c = a + 3b$
 $\Rightarrow 2c = 3b \Rightarrow b \neq 0$ [$\because c \neq 0$]

$$b + 2d = 2a + 4b \Rightarrow 2a - 2d = -3b$$

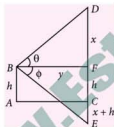
$$\therefore \frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b-\frac{3}{2}b} = -1$$

10. (a): Here $\tan \theta = \frac{x}{y}$

$$\text{and } \tan \phi = \frac{2h+x}{y}$$

$$\Rightarrow x = \frac{2h}{\cot \theta \cdot \tan \phi - 1}$$

$$\therefore CD = h + x = \frac{h \sin(\phi + \theta)}{\sin(\phi - \theta)}$$



11. (b): Consider $\frac{\sin(x_2 - x_1)}{\cos x_1 \cos x_2} + \frac{\sin(x_3 - x_2)}{\cos x_2 \cos x_3} + \dots$

$$+ \frac{\sin(x_n - x_{n-1})}{\cos x_{n-1} \cos x_n}$$

$$= \tan x_2 - \tan x_1 + \tan x_3 - \tan x_2 + \dots$$

$$+ \tan x_n - \tan x_{n-1}$$

$$= \tan x_n - \tan x_1 = \frac{\sin(x_n - x_1)}{\cos x_n \cos x_1} = \frac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$$

12. (b): $\tan^{-1} \left(\frac{\frac{a}{b+c} + \frac{b}{c+a}}{1 - \frac{a}{b+c} \cdot \frac{b}{c+a}} \right) \left(\frac{ab}{(b+c)(c+a)} < 1 \right)$

But in right angled $\triangle ABC$, $c^2 = a^2 + b^2$

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

13. (a): Given $\sin 2\beta = \sqrt{\sin \alpha \cdot \cos \alpha}$

$$\text{Now, } \cos 4\beta = 1 - 2\sin^2 2\beta = 1 - 2\sin \alpha \cdot \cos \alpha$$

$$= (\sin \alpha - \cos \alpha)^2 = 2\sin^2 \left(\alpha - \frac{\pi}{4} \right) = 2\sin^2 \left(\frac{\pi}{4} - \alpha \right)$$

14. (a): The given condition can be written as
 $(\cos^2 \alpha + \sin^2 \alpha)^3 - 3\sin^2 \alpha \cos^2 \alpha (\cos^2 \alpha + \sin^2 \alpha)$
 $+ k\sin^2 2\alpha = 1$

$$\Rightarrow \left(-\frac{3}{4} \right) \sin^2 2\alpha + k\sin^2 2\alpha = 0 \Rightarrow k = \frac{3}{4}$$

15. (d): Here $3 - 4\sin^2 x = y$

$$\therefore \sin^2 x = \frac{3-y}{4}$$

$$\text{But } 0 < \sin^2 x \leq 1 \quad (\because \sin x = 0 \Rightarrow x = n\pi)$$

$$\therefore 0 < \frac{3-y}{4} \leq 1 \Rightarrow 0 < 3-y \leq 4$$

$$\Rightarrow -1 \leq y < 3$$

16. (c): The joint equation of straight line $y = x - 1$
 and $y = -x + 1$ is $(x - y - 1)(x + y - 1) = 0$
 $\Rightarrow x^2 - y^2 - 2x + 1 = 0 \quad \dots(i)$

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Let equation of line passes through (2, 0) is

$$y = m(x - 2) \quad \dots(ii)$$

By homogenizing equation (i) with help of line (ii) is

$$x^2 - y^2 - 2x \left(\frac{mx - y}{2m} \right) + \left(\frac{mx - y}{2m} \right)^2 = 0$$

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

$$17. (a) : \text{Let } y = \frac{x^2 - x + 1}{x^2 + x + 1}$$

$$\Rightarrow yx^2 + yx + y = x^2 - x + 1$$

$$\Rightarrow (y - 1)x^2 + (y + 1)x + (y - 1) = 0, x \in \mathbb{R}$$

$$\therefore \text{Discriminant} \geq 0$$

$$\Rightarrow (y + 1)^2 - 4(y - 1) \geq 0 \Rightarrow -3y^2 + 10y - 3 \geq 0$$

$$\Rightarrow 3y^2 - 10y + 3 \leq 0 \Rightarrow (3y - 1)(y - 3) \leq 0 \Rightarrow \frac{1}{3} \leq y \leq 3$$

$$\therefore \text{Range of } y = \left[\frac{1}{3}, 3 \right]$$

$$18. (c) : \text{We must have } x^4 - 21x^2 \geq 0$$

$$\text{and } 10 - \sqrt{x^4 - 21x^2} \geq 0$$

$$\Rightarrow x^2(x^2 - 21) \geq 0 \quad \dots(i)$$

$$\text{and } 100 \geq x^4 - 21x^2 \quad \dots(ii)$$

$$(i) \text{ gives } x = 0 \text{ or } x \leq -\sqrt{21} \text{ or } x \geq \sqrt{21}$$

$$(ii) \text{ gives } x^4 - 21x^2 - 100 \leq 0$$

$$\Rightarrow (x^2 - 25)(x^2 + 4) \leq 0$$

$$\Rightarrow x^2 - 25 \leq 0 \quad (\text{as } x^2 + 4 > 0 \text{ always})$$

$$\Rightarrow -5 \leq x \leq 5$$

$\therefore$ Domain is given by $[-5, -\sqrt{21}] \cup [\sqrt{21}, 5]$ and $x = 0$.

$$19. (a) : \text{We have } \lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$$

$$= \lim_{x \rightarrow 0} a^{\sin x} \left(\frac{a^{\tan x - \sin x} - 1}{\tan x - \sin x} \right)$$

$$= \lim_{x \rightarrow 0} \left(a^{\sin x} \right) \times \lim_{t \rightarrow 0} \left(\frac{a^t - 1}{t} \right) \quad (\text{where } t = \tan x - \sin x)$$

$$= a^0 \times \log_e a = \log_e a$$

$$20. (b) : \text{Let } p = \lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2} \right) \cdot \frac{1}{3x^2}$$

$$= \frac{1}{3} \lim_{x \rightarrow 0} \frac{1+x^2 - \sqrt{1-x^2}}{x^2} \cdot \frac{1}{\sqrt{1-x^2}(1+x^2)}$$

$$= \frac{1}{3} \lim_{x \rightarrow 0} \frac{(1+x^2)^2 - (1-x^2)}{x^2} \times \frac{1}{1+x^2 + \sqrt{1-x^2}} \times \frac{1}{\sqrt{1-x^2}(1+x^2)}$$

$$= \frac{1}{3} \cdot \frac{3}{2} \cdot \frac{1}{1} = \frac{1}{2}$$

$$21. (a) : \text{Let } f(x) = a_1 \sin x + a_2 \sin 2x + \dots + a_8 \sin 8x$$

$$\text{Now, } |a_1 + 2a_2 + \dots + 8a_8| = |f'(0)| = \lim_{x \rightarrow 0} \left| \frac{f(x) - 0}{x} \right|$$

$$= \lim_{x \rightarrow 0} \left| \frac{f(x)}{\sin x} \right| \cdot \left| \frac{\sin x}{x} \right| = \lim_{x \rightarrow 0} \left| \frac{f(x)}{\sin x} \right| \leq 1$$

$$\therefore |p| \leq 1$$

$$22. (b) : \text{We have } \frac{f'(x)f(x)}{(f(x))^4 + 1} > 1$$

Integrating both sides with respect to x from a to b , we have

$$\frac{1}{2} [\tan^{-1}((f(x))^2)]_a^b > (b-a)$$

$$\Rightarrow \frac{1}{2} \left[\frac{\pi}{3} - \frac{\pi}{4} \right] > (b-a) \Rightarrow b-a < \frac{\pi}{24}$$

$$23. (a) : \text{Any point on the parabola is } (x, x^2 + 7x + 2)$$

Its distance from the line $y = 3x - 3$ is given by

$$p = \left| \frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9+1}} \right| = \left| \frac{x^2 + 4x + 5}{\sqrt{10}} \right|$$

$$= \frac{x^2 + 4x + 5}{\sqrt{10}} \quad (\because x^2 + 4x + 5 > 0 \forall x \in \mathbb{R})$$

$$\text{Now, } \frac{dp}{dx} = 0 \Rightarrow x = -2$$

$\therefore$ The required point is $(-2, -8)$.

$$24. (b) : \text{Let } P = (at_1^2, 2at_1) \text{ and } Q = (at_2^2, 2at_2)$$

Since the normals at P and Q meet on the parabola,

$$t_1 t_2 = 2.$$

Point of intersection of the normals is (x_1, y_1)

$$= (a(t_1^2 + t_2^2 + t_1 t_2 + 2), -at_1 t_2(t_1 + t_2))$$

$$\Rightarrow x_1 = a(t_1^2 + t_2^2 + t_1 t_2 + 2) = a(t_1^2 + t_2^2 + 4)$$

$$\Rightarrow a(t_1^2 + t_2^2) = x_1 - 4a \quad \dots(i)$$

$$PQ^2 = (at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2$$

$$= a^2(t_1 - t_2)^2[(t_1 + t_2)^2 + 4]$$

$$= a(t_1^2 + t_2^2 - 4)(t_1^2 + t_2^2 + 8)$$

$$= (x_1 - 8a)(x_1 + 4a) \quad [\text{From (i)}]$$

25. (b) : Since, we know that diagonals of a parallelogram intersect at midpoint.

$$\therefore \frac{a+1}{2} = \frac{a+1}{2} \text{ and } \frac{-1-a}{2} = \frac{a+1}{2}$$

$$\Rightarrow 2a = -2 \Rightarrow a = -1$$

26. (b) : Let $z = a + ib$

$$\text{Given, } z + |z| = 2 + 8i$$

$$\Rightarrow a + ib + \sqrt{a^2 + b^2} = 2 + 8i \Rightarrow b = 8, a + \sqrt{a^2 + 64} = 2$$

$$\Rightarrow a^2 + 64 = a^2 - 4a + 4 \Rightarrow -4a = 60 \Rightarrow a = -15$$

$$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

27. (a) : Since $^{10}C_0 + ^{11}C_1 + ^{12}C_2 + ^{13}C_3 + \dots + ^{999}C_{999}$
 $= ^{1000}C_{999} = ^{1000}C_{11}$

$$(\because ^{10}C_0 = ^{11}C_0 \text{ and } ^nC_r + ^nC_{r-1} = ^{n+1}C_r)$$

$$\text{So, } ^{12}C_2 + ^{13}C_3 + ^{14}C_4 + \dots + ^{999}C_{999} = ^{1000}C_{11} - 12$$

28. (d) : Let $(\sqrt{2} + 1)^6 = I + F$, where I is an integer and $0 < F < 1$.

$$\text{Let } f = (\sqrt{2} - 1)^6$$

$$\text{Now, } \sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1} \Rightarrow 0 < \sqrt{2} - 1 < 1$$

$$\Rightarrow 0 < f < 1$$

$$\begin{aligned} \text{Also, } I + F + f &= (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 \\ &= 2[{}^6C_0 \cdot 2^3 + {}^6C_2 \cdot 2^2 + {}^6C_4 \cdot 2 + {}^6C_6] \\ &= 2(8 + 60 + 30 + 1) = 198 \end{aligned}$$

Hence $F + f = 198 - I$, is an integer.

But $0 < F + f < 2$.

Therefore $F + f = 1$ and thus, $I = 197$.

29. (b) :
$$\sum_{m=1}^n {}^nC_m \left(\sum_{k=1}^m \left(\sum_{p=k}^m \frac{m!}{p!(m-p)!} \cdot \frac{p!}{k!(p-k)!} \right) \right)$$

$$= \sum_{m=1}^n {}^nC_m \left(\sum_{k=1}^m \left(\sum_{p=k}^m m-k \cdot C_{p-k} \right) \frac{m!}{k!(m-k)!} \right)$$

$$= \sum_{m=1}^n {}^nC_m \left(\sum_{k=1}^m 2^{m-k} \cdot {}^mC_k \right)$$

$$= \sum_{m=1}^n {}^nC_m \left((1+2)^m - 2^m \right) = \sum_{m=1}^n ({}^nC_m 3^m - {}^nC_m 2^m)$$

$$= (1+3)^n - 1 - (1+2)^n + 1 = 4^n - 3^n$$

30. (b) : $G_1 : 3, 6, 9, \dots, 3n$

$G_2 : 1, 4, 7, \dots, (3n-2)$

$G_3 : 2, 5, 8, \dots, (3n-1)$

Now, $a^2 - b^2$ will be divisible by 3 if a, b are selected from same group or one of them is selected from G_2 and G_3 .

$\therefore$ Total cases = ${}^nC_2 + {}^nC_2 + {}^nC_1 \cdot {}^nC_1$

$$= 3 \frac{n(n-1)}{2} + n^2$$



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GEAR UP FOR JEE MAIN 2019

Exam Dates : Between 6th to 20th January and between 6th to 20th April 2019

1. The area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$ which lies to the right of the line $x = 1$ is

(a) $\left(\frac{12-20\sqrt{3}}{2}\right)$ sq. units

(b) $\left(\frac{20-\sqrt{2}}{3}\right)$ sq. units

(c) $\left(\frac{20-12\sqrt{2}}{3}\right)$ sq. units

(d) $\left(\frac{12-20\sqrt{2}}{3}\right)$ sq. units

2. The function $f: R \rightarrow R$ defined by

$$f(x) = (x-1)(x-2)(x-3) \text{ is}$$

- (a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

3. The number of solutions of

$$\sin\left\{\sin^{-1}(\log_{1/2} x)\right\} + 2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\}\right| = 0, \text{ is}$$

- (a) 1 (b) 2
(c) 3 (d) none of these

4. If $x = \cos\theta$ and $y = \sin\theta$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

- (a) $-5y$ (b) $5y$ (c) 25 (d) $-25y$

5. Points at which the curves $\arg(z-3i) = \frac{3\pi}{4}$ and $\arg(2z-1-2i) = \frac{\pi}{4}$ intersect, is

(a) $\left(\frac{3}{4}, \frac{9}{4}\right)$ (b) $\left(\frac{5}{4}, \frac{7}{4}\right)$

(c) $\left(\frac{2}{4}, \frac{7}{4}\right)$ (d) $\left(\frac{6}{7}, \frac{4}{7}\right)$

6. Let a, b, c be the positive integers such that b/a is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b+2$, then the value of $\frac{a^2+a-14}{a+1}$ is

- (a) 2 (b) 4 (c) 6 (d) 8

7. The value of $3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ$ is

- (a) 0 (b) -1
(c) 1 (d) none of these

8. $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = \sin^{-1}\left(\frac{\sin x + \cos x}{a}\right) + c$, then $a =$

- (a) 2 (b) 3
(c) 4 (d) none of these

9. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and

$$S = \{x \in R : f(x) = f(-x)\}; \text{ then } S$$

- (a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements.

10. The tangent at a point $P(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle in two

points. The chord joining them subtends a right angle at the centre. Then, the eccentricity of the ellipse is given by

- (a) $(1 + \sin^2\theta)^{-1/2}$ (b) $(1 - \cos^2\theta)^{-1/2}$
(c) $(1 + \sin^2\theta)$ (d) $(1 + \cos^2\theta)^{1/2}$

11. If $f(x) = \begin{cases} \frac{a}{2} \frac{x^2 - x - 2}{x^2}, & x < 2 \\ b, & x = 2 \end{cases}$ is continuous at $x = 2$, then

$$\begin{cases} \frac{x - [x]}{x - 2}, & x > 2 \end{cases}$$

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

- (a) $a = 1, b = 2$ (b) $a = 1, b = 1$
 (c) $a = 2, b = 1$ (d) $a = b = 2$

12. Let r be the range of n ($\forall n \geq 1$) observations

$$x_1, x_2, \dots, x_n, \text{ if } S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}, \text{ then}$$

- (a) $S < r\sqrt{\frac{n^2+1}{n-1}}$ (b) $S \geq r\sqrt{\frac{n}{n-1}}$
 (c) $S = r\sqrt{\frac{n}{n-1}}$ (d) $S < r\sqrt{\frac{n}{n-1}}$

13. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{j} - \hat{k}$. If $\vec{b}$ is a vector satisfying $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$, then $\vec{b}$ is

- (a) $\frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$ (b) $\frac{1}{3}(5\hat{i} - 2\hat{j} - 2\hat{k})$
 (c) $3\hat{i} - \hat{j} - \hat{k}$ (d) none of these

14. The coordinates of the foot of the perpendicular drawn from the point $A(2, 4, -1)$ on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ are

- (a) $(1, -3, 4)$ (b) $(-4, 1, -3)$
 (c) $(4, 1, 3)$ (d) none of these

15. The value of a so that the volume of the parallelepiped formed by $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum is

- (a) $\frac{1}{3}$ (b) 3 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

16. The probability that a scheduled flight departs on time is 0.9, the probability that it arrives on time is 0.8 and the probability that it departs and arrives on time is 0.7. Then the probability that a plane arrives on time, given that it departs on time, is

- (a) 0.72 (b) $\frac{8}{9}$ (c) $\frac{7}{9}$ (d) 0.56

17. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $(A(\text{adj}A)A^{-1})A =$

- (a) $2 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$
 (c) $\frac{1}{6} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ (d) none of these

18. If the system of linear equations

$$\begin{aligned} x + 4ay + az &= 0 \\ x + 3by + bz &= 0 \\ x + 2cy + cz &= 0 \end{aligned}$$

have a non-trivial solution, then a, b, c are in

- (a) H.P. (b) G.P.
 (c) A.P. (d) none of these

19. The four common tangents to the ellipses

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \text{ and } \frac{x^2}{4} + \frac{y^2}{9} = 1 \text{ form}$$

- (a) a rectangle of area 13 unit²
 (b) a square of area 26 unit²
 (c) a parallelogram which is neither a square nor a rectangle
 (d) a rhombus.

20. Let $f(x)$ be a continuous in $(-\infty, \infty)$ and $f'(x)$ exists in $(-\infty, \infty)$. If $f(3) = -6$ and $f'(x) \geq 6$ for all $x \in [3, 6]$, then

- (a) $f(6) \geq 24$ (b) $f(6) \geq 12$
 (c) $f(6) \leq 24$ (d) $f(6) \leq 12$

21. A curve passes through the point $(1, \frac{\pi}{6})$.

Let the slope of the curve at each point

(x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then, the equation of the curve is

(a) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (b) $\text{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(c) $\sec\left(\frac{y}{x}\right) = \log x + 2$ (d) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

22. If $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ and $f'(x) > 0 \forall x$, then

- (a) $h(x)$ is an increasing function of x in some specified interval
 (b) $h(x)$ is an increasing function $\forall x \in R$
 (c) $h(x)$ is a decreasing function $\forall x \in R$
 (d) $h(x)$ is a decreasing function of x in some specified interval.

23. The letters of the word 'ARRANGE' is so arranged that two R do not come together. Number of such word is

- (a) 540 (b) 900
 (c) 1080 (d) none of these

24. $\int_0^{\pi} e^{\sin^2 x} \cos^3 x dx =$

- (a) 0 (b) -1 (c) 1 (d) π

25. If $f(x) = x^{\tan x}$, then $f'\left(-\frac{\pi}{6}\right) =$

(a) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}} \left\{ \frac{2\sqrt{3}}{\pi} - \frac{4}{3} \log \frac{6}{\pi} \right\}$

(b) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}} \left\{ -\frac{2\sqrt{3}}{\pi} + \frac{4}{3} \log \frac{6}{\pi} \right\}$

(c) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}} \left\{ \frac{2\sqrt{3}}{\pi} + \frac{4}{3} \log \frac{6}{\pi} \right\}$

(d) none of these

26. Electric current C , measured by a galvanometer, is given by the equation $C = k \tan \theta$, where k is a constant. Then the percentage error in the current corresponding to an error 0.7 percent in the measurement of θ when $\theta = 45^\circ$ is

(a) 1.4 (b) 2.8 (c) 1.1 (d) 2.2

27. The value of the series

$$C_0 - C_1 \frac{2^3}{2} + C_2 \frac{2^6}{3} - C_3 \frac{2^9}{4} + \dots (n+1) \text{ terms is}$$

(a) $\frac{1 - (-7)^{n+1}}{8(n+1)}$ (b) $\frac{1 - (-7)^{n+1}}{3(n+1)}$

(c) $\frac{2^n - 1}{n+1}$ (d) 0

28. The equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter is

(a) $x^2 + y^2 + 3x + y + 11 = 0$

(b) $x^2 + y^2 - 3x - y - 11 = 0$

(c) $x^2 + y^2 + 3x + y - 11 = 0$

(d) $x^2 + y^2 + 3x - y - 11 = 0$

29. Let the population of rabbits surviving at a time be governed by the differential equation

$$\frac{dp(t)}{dt} = \frac{1}{2} p(t) - 200. \text{ If } p(0) = 100, \text{ then } p(t) =$$

(a) $400 - 300e^{\frac{t}{2}}$ (b) $300 - 200e^{-\frac{t}{2}}$

(c) $600 - 500e^{\frac{t}{2}}$ (d) $400 - 300e^{-\frac{t}{2}}$

30. The equation of the plane passing through the points $(0, 1, 0)$ and $(3, 4, 1)$ and parallel to the line

$$\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5} \text{ is}$$

(a) $4x - 13y + 15z - 13 = 0$

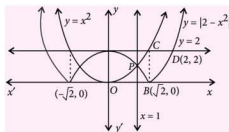
(b) $8x - 13y + 15z + 15 = 0$

(c) $8x - 13y + 15z + 13 = 0$

(d) none of these

SOLUTIONS

1. (c) : Let A be the required area.



Then, $A = \text{area } PBCP + \text{area } BCD$

$$\Rightarrow A = \int_{-1}^{\sqrt{2}} \{x^2 - (2 - x^2)\} dx + \int_{\sqrt{2}}^2 \{2 - (x^2 - 2)\} dx$$

$$\Rightarrow A = \int_{-1}^{\sqrt{2}} (2x^2 - 2) dx + \int_{\sqrt{2}}^2 (4 - x^2) dx$$

$$\Rightarrow A = \left[\frac{2}{3} x^3 - 2x \right]_{-1}^{\sqrt{2}} + \left[4x - \frac{x^3}{3} \right]_{\sqrt{2}}^2$$

$$\Rightarrow A = \left(\frac{20 - 12\sqrt{2}}{3} \right) \text{ sq. units}$$

2. (b) : The given function $f: R \rightarrow R$ is defined by $f(x) = (x-1)(x-2)(x-3)$

Since, 1, 2, 3 $\in R$ and $f(1) = 0, f(2) = 0, f(3) = 0$

Now, $f(1) = f(2)$ but $1 \neq 2$

$\therefore f$ is not one-one mapping.

Again if $y = (x-1)(x-2)(x-3)$, then $x^3 - 6x^2 + 11x - 6 - y = 0$ and any third degree equation must have at least one real root. Hence it is an onto mapping.

3. (d) : We have,

$$\sin \left\{ \sin^{-1}(\log_{1/2} x) \right\} + 2 \left| \cos \left\{ \sin^{-1} \left(\frac{x}{2} - \frac{3}{2} \right) \right\} \right| = 0$$

The two terms on LHS of the given equation are

$$\text{meaningful, if } -1 \leq \log_{1/2} x \leq 1 \text{ and } -1 \leq \frac{x}{2} - \frac{3}{2} \leq 1$$

$$\Rightarrow 2 \geq x \geq \frac{1}{2} \text{ and } 1 \leq x \leq 5 \Rightarrow 1 \leq x \leq 2$$

Now, $1 \leq x \leq 2$

$$\Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 1 \Rightarrow -\frac{1}{2} \leq \frac{x}{2} - 1 \leq 0$$

$$\Rightarrow -\frac{\pi}{6} \leq \sin^{-1} \left(\frac{x}{2} - 1 \right) \leq 0$$

$$\Rightarrow \frac{\sqrt{3}}{2} \leq \cos \left\{ \sin^{-1} \left(\frac{x}{2} - 1 \right) \right\} \leq 1$$

$$\sqrt{3} \leq 2 \cos \left\{ \sin^{-1} \left(\frac{x}{2} - 1 \right) \right\} \leq 2$$

$$\Rightarrow \sqrt{3} \leq 2 \left| \cos \left\{ \sin^{-1} \left(\frac{x}{2} - 1 \right) \right\} \right| \leq 2 \quad \dots(i)$$

Again, $1 \leq x \leq 2$

$$\Rightarrow -1 \leq \log_{1/2} x \leq 0 \Rightarrow -\frac{\pi}{2} \leq \sin^{-1}(\log_{1/2} x) \leq 0$$

$$\Rightarrow -1 \leq \sin \{ \sin^{-1}(\log_{1/2} x) \} \leq 0 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\sqrt{3} - 1 \leq \sin \{ \sin^{-1}(\log_{1/2} x) \} + 2 \left| \cos \left\{ \sin^{-1} \left(\frac{x}{2} - 1 \right) \right\} \right| \leq 2$$

$$\Rightarrow \sin \{ \sin^{-1}(\log_{1/2} x) \} + 2 \left| \cos \left\{ \sin^{-1} \left(\frac{x}{2} - 1 \right) \right\} \right| \neq 0$$

for any x . Hence the given equation has no solution.

4. (d): We have, $x = \cos \theta$ and $y = \sin 5\theta$

$$\therefore \frac{dx}{d\theta} = -\sin \theta \text{ and } \frac{dy}{d\theta} = 5 \cos 5\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{5 \cos 5\theta}{\sin \theta} \quad \dots(i)$$

$$\text{Now, } \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\begin{aligned} &= \frac{d}{dx} \left(-\frac{5 \cos 5\theta}{\sin \theta} \right) = -5 \frac{d}{d\theta} \left(\frac{\cos 5\theta}{\sin \theta} \right) \frac{d\theta}{dx} \\ &= -5 \times \frac{-5 \sin 5\theta \sin \theta - \cos 5\theta \cos \theta}{\sin^2 \theta} \times \frac{-1}{\sin \theta} \\ &= \frac{-25 \sin 5\theta \sin \theta - 5 \cos \theta \cos 5\theta}{\sin^3 \theta} \quad \dots(ii) \end{aligned}$$

Thus, from (i) and (ii), we have

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = \sin^2 \theta \times \frac{-25 \sin 5\theta \sin \theta - 5 \cos \theta \cos 5\theta}{\sin^3 \theta} - \cos \theta \left(-\frac{5 \cos 5\theta}{\sin \theta} \right)$$

$$= \frac{-25 \sin 5\theta \sin \theta - 5 \cos \theta \cos 5\theta + 5 \cos \theta \cos 5\theta}{\sin \theta}$$

$$= \frac{-25 \sin 5\theta \sin \theta}{\sin \theta} = -25 \sin 5\theta = -25y$$

$$\text{Therefore, } (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = -25y$$

5. (b): Let $z = x + iy$

$$\therefore z - 3i = x + iy - 3i = x + i(y-3)$$

$$\text{Now, } \arg(x + i(y-3)) = \frac{3\pi}{4} \Rightarrow \tan^{-1} \left(\frac{y-3}{x} \right) = \frac{3\pi}{4}$$

$$\Rightarrow \frac{(y-3)}{x} = -1 \Rightarrow x + y = 3 \quad \dots(i)$$

$$\begin{aligned} \text{Again, } 2z - 1 - 2i &= 2(x + iy) - 1 - 2i \\ &= (2x-1) + i(2y-2) \end{aligned}$$

$$\therefore \arg(2z - 1 - 2i) = \frac{\pi}{4} \Rightarrow \arg \{ (2x-1) + i(2y-2) \} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{2y-2}{2x-1} \right) = \frac{\pi}{4} \Rightarrow \frac{2y-2}{2x-1} = 1$$

$$\Rightarrow 2x - 2y = -1 \quad \dots(ii)$$

$$\text{Solving (i) and (ii), we get } x = \frac{5}{4}, y = \frac{7}{4}$$

6. (b): It is given that a, b, c are in G.P., and the A.M. of a, b, c is $b + 2$.

$$\therefore b^2 = ac \text{ and } \frac{a+b+c}{3} = b+2$$

$$\Rightarrow b^2 = ac \text{ and } a + c = 2b + 6$$

$$\Rightarrow c = \frac{b^2}{a} \text{ and } a - 2b + c = 6$$

$$\Rightarrow a - 2b + \frac{b^2}{a} = 6 \Rightarrow \left(\frac{b}{a} \right)^2 - 2 \left(\frac{b}{a} \right) + 1 = \frac{6}{a}$$

$$\Rightarrow \left(\frac{b}{a} - 1 \right)^2 = \frac{6}{a}$$

SOLUTION SENDERS

Samurai Sudoku (October)

- Sourabh Mondal (West Bengal)

Samurai Sudoku (September)

- Anoushka Patra (West Bengal)
- Akanksha Das (West Bengal)
- Bharat Kumar (Rajasthan)

Mathdoku (September)

- Anoushka Patra (West Bengal)
- G. Lakshmi Prasanna (Telangana)

$\Rightarrow \frac{6}{a}$ is a perfect square of integer

$$\left[\because \frac{b}{a} \text{ is an integer} \right]$$

$$\Rightarrow a = 6 \quad \therefore \frac{a^2 + a - 14}{a + 1} = \frac{36 + 6 - 14}{6 + 1} = 4$$

7. (c) : We know that

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$\Rightarrow \tan 30^\circ = \frac{3 \tan 10^\circ - \tan^3 10^\circ}{1 - 3 \tan^2 10^\circ} \quad (\text{Putting } A = 10^\circ)$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3 \tan 10^\circ - \tan^3 10^\circ}{1 - 3 \tan^2 10^\circ}$$

$$\Rightarrow 1 - 3 \tan^2 10^\circ = \sqrt{3}(3 \tan 10^\circ - \tan^3 10^\circ)$$

$$\Rightarrow 1 + 9 \tan^4 10^\circ - 6 \tan^2 10^\circ = 3(9 \tan^2 10^\circ + \tan^6 10^\circ - 6 \tan^4 10^\circ)$$

$$\Rightarrow 3 \tan^6 10^\circ - 27 \tan^4 10^\circ + 33 \tan^2 10^\circ = 1$$

8. (b) : Here, $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx$

$$= \int \frac{1}{\sqrt{3^2 - (\sin x + \cos x)^2}} d(\cos x + \sin x)$$

$$= \sin^{-1} \left(\frac{\sin x + \cos x}{3} \right) + c$$

9. (a) : Here, the given equation is

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$$

Now replacing x to $1/x$ in the above equation, we get

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$$

Eliminating $f\left(\frac{1}{x}\right)$ from the above two equations, we get

$$3f(x) = \frac{6}{x} - 3x \Rightarrow f(x) = \frac{2}{x} - x$$

Now to get the elements of S , we solve $f(x) = f(-x)$

$$\Rightarrow \frac{2}{x} - x = -\frac{2}{x} + x \Rightarrow \frac{2}{x} - x = 0$$

$$\Rightarrow x^2 = 2 \Rightarrow x = \pm \sqrt{2}$$

10. (a) : The equation of the tangent at

$P(a \cos \theta, b \sin \theta)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \quad \dots(i)$$

The combined equation of the chords joining the points of intersection of (i) and the auxiliary circle $x^2 + y^2 = a^2$ is

$$x^2 + y^2 = a^2 \left(\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta \right)^2$$

The lines given by this equation are at right angle.

$\therefore$ Coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow 1 - \cos^2 \theta + 1 - a^2 \left(\frac{\sin^2 \theta}{b^2} \right) = 0$$

$$\Rightarrow \sin^2 \theta + 1 - \frac{a^2}{b^2} \sin^2 \theta = 0$$

$$\Rightarrow \sin^2 \theta \left(1 - \frac{a^2}{b^2} \right) + 1 = 0$$

$$\Rightarrow -\sin^2 \theta (a^2 - b^2) + b^2 = 0$$

$$\Rightarrow -a^2 e^2 \sin^2 \theta + a^2 (1 - e^2) = 0$$

$$\Rightarrow 1 = e^2 (1 + \sin^2 \theta) \Rightarrow e = (1 + \sin^2 \theta)^{-1/2}$$

11. (b) : We know that,

$$x^2 - x - 2 = \begin{cases} (x+1)(x-2) < 0, & \text{if } x \in (-1, 2) \\ (x+1)(x-2) > 0, & \text{if } x \in (2, \infty) \end{cases}$$

Thus the function can be redefined as

$$f(x) = \begin{cases} a & \text{if } x < 2 \\ b & \text{if } x = 2 \\ 1 & \text{if } x > 2 \end{cases}$$

(Since in the neighbourhood of 2 on left side

$$|x^2 - x - 2| = -(x^2 - x - 2) \text{ and } x - [x] = x - 2 \text{ if } x \in (2, 3))$$

$\therefore f(x)$ is continuous at $x = 2 \Rightarrow a = b = 1$

12. (d) : Here range = r = largest value - smallest value
 $= \text{Max} |x_i - x_j| (i \neq j)$

$$\text{And } S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\text{Now, } (x_i - \bar{x})^2 = \left[x_i - \frac{x_1 + x_2 + \dots + x_n}{n} \right]^2$$

$$= \frac{1}{n^2} [(x_i - x_1) + (x_i - x_2) + \dots + (x_i - x_n)]^2$$

$$= \frac{1}{n^2} [(x_i - x_1) + (x_i - x_2) + \dots + (x_i - x_{i-1}) + (x_i - x_{i+1}) + \dots + (x_i - x_n)]^2$$

$$\Rightarrow (x_i - \bar{x})^2 \leq \frac{1}{n^2} [(n-1)r]^2 \quad [\because |x_i - x_j| \leq r]$$

$$\Rightarrow \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \leq \frac{1}{n^2(n-1)} \sum [(n-1)r]^2$$

(Summing up and dividing by $(n-1)$ both sides)

$$\begin{aligned} &= \frac{1}{n^2} \cdot \frac{1}{n-1} n(n-1)^2 r^2 \\ &= \frac{n-1}{n} r^2 \\ &< \frac{n}{n-1} r^2 \quad \left(\because \forall n > 1, n > \frac{1}{n} \right) \end{aligned}$$

Therefore, $S^2 < \frac{n}{n-1} \cdot r^2$ or $S < r \sqrt{\frac{n}{n-1}}$

13. (a) : Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $\vec{a} \cdot \vec{b} = 3 \Rightarrow x + y + z = 3$, and

$\vec{a} \times \vec{b} = \vec{c} \Rightarrow (z-y)\hat{i} + (x-z)\hat{j} + (y-x)\hat{k} = \hat{j} - \hat{k}$

$\Rightarrow z-y=0, x-z=1$ and $y-x=-1$

Solving these equations with $x+y+z=3$, we get

$$x = \frac{5}{3}, y = \frac{2}{3} \text{ and } z = \frac{2}{3}$$

Therefore, $\vec{b} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

14. (b) : Let the foot of the perpendicular be L .

The general point on the straight line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} \text{ is}$$

$$(\lambda-5, 4\lambda-3, -9\lambda+6) \quad \dots(i)$$

Let $L \equiv (\lambda-5, 4\lambda-3, -9\lambda+6)$

$\therefore$ The direction ratios of AL are

$$\lambda-7, 4\lambda-7, -9\lambda+7$$

Since AL is perpendicular to the line (i), therefore

$$1(\lambda-7) + 4(4\lambda-7) - 9(-9\lambda+7) = 0$$

$$\Rightarrow \lambda = 1$$

Thus, $L \equiv (\lambda-5, 4\lambda-3, -9\lambda+6) = (-4, 1, -3)$

15. (c) : Let V be the volume of the parallelepiped formed by the given vectors. Let $\vec{\alpha} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{\beta} = \hat{j} + a\hat{k}$ and $\vec{\gamma} = a\hat{i} + \hat{k}$.

$$\text{Then, } V = [\vec{\alpha} \vec{\beta} \vec{\gamma}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a$$

Clearly, V will be minimum when $1 + a^3 - a$ is minimum.

$$\text{Now, } V = 1 + a^3 - a \Rightarrow \frac{dV}{da} = 3a^2 - 1 \text{ and } \frac{d^2V}{da^2} = 6a$$

For maximum or minimum value of V , we must have

$$\frac{dV}{da} = 0 \Rightarrow a = \pm \frac{1}{\sqrt{3}}$$

$$\text{Clearly, } \left(\frac{d^2V}{da^2} \right)_{a=\frac{1}{\sqrt{3}}} = \frac{6}{\sqrt{3}} > 0$$

Hence, V is minimum for $a = \frac{1}{\sqrt{3}}$.

16. (c) : Let E_1 be the event that the flight departs on time, E_2 be the event that the flight arrives on time.

$$\therefore P(E_1) = 0.9, P(E_2) = 0.8 \text{ and } P(E_1 \cap E_2) = 0.7$$

$$\text{Now, } P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{0.7}{0.9} = \frac{7}{9}$$

17. (a) : We have, $|A| = \begin{vmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0 + 7 - 1 = 6$

$$\therefore (A(\text{adj}A)A^{-1})A = (A(\text{adj}A))(A^{-1}A)$$

$$\Rightarrow (A(\text{adj}A)A^{-1})A = (|A|)I$$

$$\Rightarrow (A(\text{adj}A)A^{-1})A = |A|I = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 2 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

18. (a) : The given system of linear equations has a non-trivial solution. Therefore,

$$\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 4a-3b & a-b \\ 0 & 3b-2c & b-c \\ 1 & 2c & c \end{vmatrix} = 0$$

$$[\text{Applying } R_1 \rightarrow R_1 - R_2 \text{ and } R_2 \rightarrow R_2 - R_3]$$

$$\Rightarrow (4a-3b)(b-c) - (3b-2c)(a-b) = 0$$

$$\Rightarrow ab - 2ac + bc = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b} \Rightarrow a, b, c \text{ are in H.P.}$$

MPP-7 CLASS XII ANSWER KEY

1. (d) 2. (b) 3. (a) 4. (c) 5. (d)
6. (d) 7. (c) 8. (a,c,d) 9. (b,d) 10. (a,c,d)
11. (a,c) 12. (a,c) 13. (b,c,d) 14. (b) 15. (d)
16. (a) 17. (6) 18. (9997) 19. (314) 20. (3)

19. (b): Any tangent to

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \text{ is } y = mx + \sqrt{9m^2 + 4} \quad \dots(i)$$

It touches $\frac{x^2}{4} + \frac{y^2}{9} = 1$. So, (i) is of the form

$$x = m'y + \sqrt{9m'^2 + 4} \quad \dots(ii)$$

$$\text{Comparing (i) and (ii), } \frac{m}{-1} = \frac{-1}{m'} = \frac{\sqrt{9m^2 + 4}}{\sqrt{9m'^2 + 4}}$$

$$\Rightarrow mm' = 1 \text{ and } m^2 = \frac{9m'^2 + 4}{9m'^2 + 4}$$

$$\Rightarrow m^2(9m'^2 + 4) = 9m'^2 + 4$$

$$\Rightarrow 9 + 4m^2 = 9m'^2 + 4 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1 \quad (\because mm' = 1)$$

Therefore the common tangents are

$$y = x \pm \sqrt{13} \text{ and } y = -x \pm \sqrt{13}$$

Clearly the above 4 lines form a square of sides $\sqrt{26}$ units i.e., of area 26 square units.

20. (b): Given that $f(x)$ is continuous in $(-\infty, \infty)$ and $f'(x)$ exists in $(-\infty, \infty)$.

$$\therefore f'(c) = \frac{f(6) - f(3)}{6 - 3}, \text{ where } c \in (3, 6)$$

$$\Rightarrow \frac{f(6) - f(3)}{6 - 3} \geq 6 \Rightarrow f(6) - f(3) \geq 18 \Rightarrow f(6) \geq 12$$

$$[\because f(3) = -6 \text{ and } f'(x) \geq 6 \Rightarrow f'(c) \geq 6]$$

21. (a): Given that

$$\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0 \quad \dots(i)$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$(i) \text{ becomes, } v + x \frac{dv}{dx} = v + \sec v$$

$$\Rightarrow \cos v dv = \frac{1}{x} dx$$

Integrating both sides, we get

$$\sin v = \log x + c \Rightarrow \sin\left(\frac{y}{x}\right) = \log x + c \quad \dots(ii)$$

$$\text{It passes through the point } \left(1, \frac{\pi}{6}\right).$$

$$\text{Therefore } \sin \frac{\pi}{6} = \log 1 + c \Rightarrow c = \frac{1}{2}$$

$$\text{Now (ii) becomes, } \sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}.$$

$$22. (b): \because h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$$

$$\therefore h'(x) = f'(x) - 2f(x)f'(x) + 3\{f(x)\}^2 f'(x)$$

$$= 3f'(x) \left[\{f(x)\}^2 - 2f(x) \cdot \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \frac{1}{3} - \frac{1}{9} \right]$$

$$= 3f'(x) \left[\left\{f(x) - \frac{1}{3}\right\}^2 + \frac{2}{9} \right]$$

By question, $f'(x) > 0$

Thus $h'(x) > 0$ for all real values of x .

Therefore $h(x)$ is an increasing function for all real values of x .

23. (b): The word ARRANGE is consist of 7 letters. Among these 7 letters there are two R and two A. Considering the two letters (RR) as a single letter, the given word can be arranged in $\frac{6!}{2!}$ ways, where two R are always together.

$\therefore$ The number of words where two R do not come together are $\frac{7!}{2!2!} - \frac{6!}{2!} = \frac{6!}{2!} \left(\frac{7}{2} - 1 \right) = \frac{6!}{2!} \cdot \frac{5}{2} = 900$.

$$24. (a): \text{ Let } I = \int_0^{\frac{\pi}{2}} e^{\sin^2 x} \cos^3 x dx$$

$$= \int_0^{\frac{\pi}{2}} [e^{\sin^2 x} \cos^3 x + e^{\sin^2(\pi-x)} \cos^3(\pi-x)] dx$$

$$\left[\because \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(2a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} [e^{\sin^2 x} \cos^3 x + e^{\sin^2 x} \cos^3 x] dx = 0$$

25. (b): We have $f(x) = |x|^{\tan x}$

$$\Rightarrow f(x) = (-x)^{-\tan x} [\because \text{In the neighbourhood of}$$

$$x = -\frac{\pi}{6}, |x| = -x \text{ and } |\tan x| = -\tan x]$$

$$\text{Now, } \log f(x) = -\tan x \log(-x)$$

Differentiating both sides w.r.t. x , we get

$$\frac{1}{f(x)} \cdot f'(x) = \left[-\sec^2 x \cdot \log(-x) - \frac{\tan x}{x} \right]$$

$$\Rightarrow f'(x) = (-x)^{-\tan x} \left[-\sec^2 x \cdot \log(-x) - \frac{\tan x}{x} \right]$$

$$\Rightarrow f'\left(-\frac{\pi}{6}\right) = \left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}} \left\{ -\frac{4}{3} \log \frac{\pi}{6} - \frac{2\sqrt{3}}{\pi} \right\}$$

26. (c) : Here, we have

$$C = k \tan \theta \Rightarrow \frac{dC}{d\theta} = k \sec^2 \theta \Rightarrow \frac{dC}{d\theta} \delta \theta = k \sec^2 \theta \delta \theta$$

$$\Rightarrow \delta C = k \sec^2 \theta \delta \theta \quad \left[\because \delta C = \frac{dC}{d\theta} \delta \theta \right]$$

$$\text{Now, } \frac{\delta C}{C} \times 100 = \frac{k \sec^2 \theta \delta \theta}{k \tan \theta} \times 100 = \frac{\sec^2 \theta}{\tan \theta} \delta \theta \times 100$$

$$= 2 \times \frac{\pi}{4} \cdot \frac{\delta \theta}{\theta} \times 100 = \frac{\pi}{2} \times 0.7 = 1.57 \times 0.7 = 1.099$$

$\therefore$ Percentage of error in C is 1.1.

27. (a) : Consider

$$(1-x^3)^n = C_0 - C_1 x^3 + C_2 x^6 - C_3 x^9 + \dots$$

$$\Rightarrow x^2(1-x^3)^n = C_0 x^2 - C_1 x^5 + C_2 x^8 - C_3 x^{11} + \dots$$

Now integrating both sides w.r.t. x from limits 0 to 2, we get

$$\int_0^2 x^2(1-x^3)^n dx = \left[C_0 \cdot \frac{x^3}{3} - C_1 \cdot \frac{x^6}{6} + C_2 \cdot \frac{x^9}{9} - \dots \right]_0^2$$

$$= \left(C_0 \cdot \frac{2^3}{3} - C_1 \cdot \frac{2^6}{6} + C_2 \cdot \frac{2^9}{9} - \dots \right) \quad \dots(i)$$

$$\text{Now, in L.H.S. put } 1-x^3 = z \Rightarrow x^2 dx = -\frac{1}{3} dz$$

$$\therefore \text{ L.H.S.} = \int z^n \left(-\frac{1}{3} \right) dz = -\frac{1}{3} \int z^n dz = -\frac{1}{3} \frac{z^{n+1}}{n+1} \Bigg|_{-7}^1$$

$$= \frac{1}{3(n+1)} - \frac{1}{3} \frac{(-7)^{n+1}}{n+1} = \frac{1}{3(n+1)} (1 - (-7)^{n+1})$$

$$\text{Now (i) becomes } \frac{1}{3(n+1)} (1 - (-7)^{n+1})$$

$$= C_0 \cdot \frac{2^3}{3} - C_1 \cdot \frac{2^6}{6} + C_2 \cdot \frac{2^9}{9} - \dots$$

Multiplying L.H.S and R.H.S by $\frac{3}{2^3}$, we get

$$C_0 - C_1 \frac{2^3}{2} + C_2 \frac{2^6}{3} - \dots (n+1) \text{ terms} = \frac{1 - (-7)^{n+1}}{8(n+1)}$$

28. (c) : Let the line $3x + y + 5 = 0$ cuts the circle $x^2 + y^2 = 16$ at the points A and B , hence, $\overline{AB}$ is a chord of the circle. The mid-point of the chord

$\overline{AB}$ is $\left(-\frac{3}{2}, -\frac{1}{2} \right)$ (the foot of the perpendicular from the centre to the chord) and length of the chord

$$\overline{AB} = 2 \times \sqrt{16 - \frac{10}{4}} = 2\sqrt{\frac{27}{2}}$$

$\therefore$ Centre of the circle, whose diameter is AB , is

$$\left(-\frac{3}{2}, -\frac{1}{2} \right) \text{ and its radius} = \frac{1}{2} \overline{AB} = \sqrt{\frac{27}{2}}$$

Thus the equation of the circle is

$$\left(x + \frac{3}{2} \right)^2 + \left(y + \frac{1}{2} \right)^2 = \frac{27}{2} \Rightarrow x^2 + y^2 + 3x + y = 11.$$

29. (a) : We have, $\frac{dp(t)}{dt} = \frac{1}{2} p(t) - 200$

$$\Rightarrow \frac{dp(t)}{dt} = \frac{p(t) - 400}{2} \Rightarrow \int \frac{dp(t)}{p(t) - 400} = \frac{1}{2} \int dt$$

$$\Rightarrow \log |p(t) - 400| = \frac{1}{2} t + \log c$$

$$\Rightarrow \log \left| \frac{p(t) - 400}{c} \right| = \frac{1}{2} t \Rightarrow p(t) = ce^{\frac{t}{2}} + 400$$

When $p(0) = 100$, we get,

$$100 = ce^{\frac{0}{2}} + 400 \Rightarrow c = -300$$

$\therefore$ The required solution is $p(t) = 400 - 300e^{\frac{t}{2}}$

30. (c) : The equation of the plane passing through the point $(0, 1, 0)$ is

$$ax + b(y-1) + cz = 0 \quad \dots(i)$$

Since (i) is also passing through the point $(3, 4, 1)$

$$\therefore 3a + 3b + c = 0 \quad \dots(ii)$$

Again the plane (i) is parallel to the line

$$\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$$

$$\therefore 2a + 7b + 5c = 0 \quad \dots(iii)$$

Now solving (ii) and (iii), we get

$$\frac{a}{15-7} = \frac{b}{2-15} = \frac{c}{21-6} \Rightarrow \frac{a}{8} = \frac{b}{-13} = \frac{c}{15} = \lambda$$

$$\therefore a = 8\lambda, b = -13\lambda, c = 15\lambda$$

Now (i) becomes

$$8\lambda x - 13\lambda y + 15\lambda z = -13\lambda$$

$$\Rightarrow 8x - 13y + 15z + 13 = 0$$



COMIC CAPSULE

The Romans didn't find algebra very challenging, because X was always 10.

HOW TO PREPARE FOR

JEE Main in 60 DAYS?



Anxiety starts to build up among aspirants as time progresses towards the Exam. To deal with this, students should make week-wise preparation planning prior to the exam date.

Read on the article to sail smoothly through these 60 days

First of all review the test basics like syllabus, length of the test paper, duration of test, test dates, etc.

Refer to some last-minute preparatory books for your JEE main like crash course, previous years' question bank, mock test papers, etc. Take few full-length practice tests from the book or online to check your readiness. Keep record of your scores and time you'd take during each test. Make a time management chart for 2 months. Make sure that the entire syllabus needs to be revised within 8 weeks and save rest of the days for attempting practice papers and revise important formulas and facts again.

1. Begin with your weakest subject as demonstrated by the baseline score.
- ✓ Learn and revise the syllabus of that subject chapterwise along with practice questions. MTG Books like JEE Main Guides and 17 Years' JEE Main Chapterwise Solutions will really help you at this stage.
- ✓ Keep a check of your mistakes while attempting chapterwise practice questions by correcting your methods.
- ✓ Once you revise the entire syllabus take another full-length test paper of the subject you revised. Check the level of improvement from baseline score.
- ✓ Fine tune by going over questions you marked wrong in the test to check what kind of mistakes you are making. Revise the topics until the concepts become clear to you.

2. Now move on to next weak subject and follow the same steps as you did for the previous subject.

3. Move on to strongest subject. Revise the topics in which you scored less in your full-length tests which you attempted initially.

Now you have revised all the three subjects properly so, it is the time to go through short revision notes like flow chart or formula box, etc. for your last-minute preparation just few days before the Exam.

- ✓ Take a full-length practice test online in exam like environment as much as possible. For online practice tests visit www.test.pcmbtoday.com or download the app **MTG Champion** and attempt all chapterwise and full-length tests available there. You will get a complete performance report which will help you to check your readiness for JEE.
- ✓ Grade your practice test and cross-check every wrong answer with the explanation for your wrong answer. Determine what you've missed and what you need to do to improve. Go over the sections where you missed the most.
- ✓ Take another full-length practice test, simulating the testing environment. Again, go through every missed problem, looking for weak areas. Go back to the book and see if you can improve by yourself. Still need help in certain topics, take help from your friends and teachers.
- ✓ Go back through weakest subject and work through the problems again, memorizing testing strategies, reviewing practice problems.

Be positive and follow the given tips

- ☺ Eat healthy food, drink lot of water and get plenty of sleep.
- ☺ Engage in some fun filled activity or listen to your favourite music to help you relax.
- ☺ Keep attempting online full-length tests, they help you to check your speed so that you can train yourself to finish the test in 3 hours as in JEE Main.
- ☺ Check the information bulletin to reconfirm the date, timings, disciplines to be followed at the exam hall and carefully read the Guidelines to Candidates for Computer Based Examination for JEE Main.
- ☺ Pack your stationary like pencils, eraser, sharpener, ball pens and Hall ticket, night before exam.
- ☺ Now Relax. You are prepared for your test, and completely ready for the D-Day!!

Challenging PROBLEMS



Probability



1. If $x, y, z \in R$ and $x + y + z = 5$, $xy + yz + zx = 3$, probability for x to be positive only is

- (a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{13}{16}$ (d) $\frac{15}{16}$

2. If F is the set of all onto functions from a set of vowels to set having 3 elements and $f \in F$ is chosen randomly, then the probability that $f^{-1}(x)$ is a singleton is

- (a) $\frac{7}{15}$ (b) $\frac{8}{15}$ (c) $\frac{9}{15}$ (d) $\frac{10}{15}$

3. Probability that a random chosen three digit number has exactly 3 factors is

- (a) $\frac{2}{225}$ (b) $\frac{7}{900}$ (c) $\frac{1}{300}$ (d) $\frac{4}{900}$

4. Six persons stand at random in a queue for buying cinema tickets. Individually three of them have only a fifty rupee note each while each of the other three have a hundred rupee note only. The booking clerk has an empty cash box, probability that six persons get tickets without waiting for change is, (cost of one ticket is ₹50 and each person gets one ticket only)

- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

5. A number is chosen at random from the set of all 4-digit numbers each of which contains not more than 2 different digits, probability that it does not contain the digit zero is

- (a) $\frac{7}{64}$ (b) $\frac{37}{64}$ (c) $\frac{47}{64}$ (d) $\frac{57}{64}$

6. Two persons A and B are throwing 3 dice taking turns. If A throws 8, then the probability that B throws a higher number is

- (a) $\frac{5}{27}$ (b) $\frac{9}{17}$ (c) $\frac{8}{27}$ (d) $\frac{20}{27}$

7. Each face of a cubical die is numbered with a distinct number from among the first six odd numbers; such that the sum of the two numbers on any pair of opposite face is 12, if ten such dies are thrown

simultaneously, then find the probability that the sum of the numbers that turn up is exactly 53.

- (a) $\frac{53}{6^{10}}$ (b) $\frac{153}{6^{10}}$ (c) $\frac{3}{6^{10}}$ (d) 0

8. If three numbers are chosen randomly from the set $\{1, 3, 3^2, \dots, 3^n\}$ without replacement, then the probability that they form an increasing geometric progression is

- (a) $\frac{3}{2n}$ if n is odd (b) $\frac{3}{2n}$ if n is even

- (c) $\frac{3n}{n^2 - 1}$ if n is even (d) $\frac{3n}{2(n^2 - 1)}$ if n is odd

9. Suppose $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are chosen respectively by throwing a die three times. The probability that $f(x)$ is an increasing function is

- (a) $\frac{4}{9}$ (b) $\frac{3}{8}$ (c) $\frac{2}{5}$ (d) $\frac{8}{17}$

10. A box contains 5 pairs of shoes. If 6 shoes are selected at random, the probability that exactly one pair of shoes is obtained is

- (a) $\frac{4}{21}$ (b) $\frac{2}{21}$ (c) $\frac{8}{21}$ (d) $\frac{16}{21}$

11. An unbiased coin is tossed 12 times. The probability that at least 7 consecutive heads show up is

- (a) $\frac{1}{128}$ (b) $\frac{1}{64}$ (c) $\frac{9}{256}$ (d) $\frac{7}{256}$

12. Two integers x and y are chosen from the set $\{0, 1, 2, 3, \dots, 2n\}$, with replacement, the probability that $|x - y| \leq n$ ($n \in N$) is

- (a) $\frac{3n^2 + 3n + 1}{(2n + 1)^2}$ (b) $\frac{3n^2 + 3n}{(2n + 1)^2}$

- (c) $\frac{3n^2 + 1}{(2n + 1)^2}$ (d) $\frac{n^2 + n + 1}{(2n + 1)^2}$

By : Tapas Kr. Yogi, Visakhapatnam Mob : 99533632105

MATHEMATICS TODAY | NOVEMBER '18

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13. If the third power of a natural number ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

- (a) $\frac{3}{10}$ (b) $\frac{9}{10}$ (c) $\frac{4}{9}$ (d) $\frac{3}{4}$

14. The sum of two natural numbers is 30. The probability that their product is less than 150 is

- (a) $\frac{1}{5}$ (b) $\frac{3}{29}$ (c) $\frac{1}{6}$ (d) $\frac{6}{29}$

15. An unbiased coin is tossed n times. Then the expected value of $|H - T|$, where H is number of heads and T is number of tails, is

- (a) $\frac{n}{2^{n-1}} \left[\binom{n-1}{2} \right]$ (b) $\frac{n+1}{2^{n-1}} \left[\binom{n}{2} \right]$
 (c) $\frac{n}{2^{n-1}} \left[\binom{n}{2} \right]$ (d) $\frac{n}{2^{n-1}} \left[\binom{n+1}{2} \right]$

SOLUTIONS

1. (c) : Range of x is $\left[-1, \frac{13}{3}\right]$

So, probability of x to be positive is

$$\frac{\int_{\frac{0}{3}}^{\frac{13}{3}} dx}{\int_{-1}^{\frac{13}{3}} dx} = \frac{\frac{13}{3}}{\frac{13}{3} + 1} = \frac{13}{16}$$

2. (a) : Number of onto functions from set A having 5 elements to set B having 3 elements is 150 we shall now count onto functions which satisfy $f^{-1}(x)$ is singleton. We can choose a singleton in ${}^5C_1$ ways. The remaining 4 elements can be mapped to remaining 2 elements in $2^4 - 2 = 14$ ways.

$$\therefore \text{Required probability} = \frac{5(14)}{150} = \frac{7}{15}$$

3. (b) : A number has exactly 3 factors if the number is square of a prime number. Squares of 11, 13, 17, 19, 23, 29, 31 are 3 digit numbers.

$$\therefore \text{Required probability} = \frac{7}{900}$$

4. (c) : Here random experiment of arranging 6 persons in a line is $n(S) = 6! = 720$

Let 'a' denotes the person having ₹50 note and 'b' denotes the person having ₹100 note each since all the six persons, should get ticket. First place should be occupied by a person having ₹50 note and sixth place should be occupied by a person having ₹100 note. So, possible cases are

a	aa	bb	b
a	ab	ab	b
a	ab	ba	b
a	ba	ab	b
a	ba	ba	b

'a's can arrange among themselves and 'b's can arrange among themselves in $n(E) = 3!3! + 3!3! + 3!3! + 3!3! + 3!3! = 180$ ways

$$\therefore \text{Required probability} = \frac{180}{720} = \frac{1}{4}$$

5. (d) : If $a \neq 0$ the numbers with 0 and a are $a000, a00a, a0a0, aa00, aa0a, a0aa$. These are $9 \times 7 = 63$

Now, if $a \neq b$, $ab \neq 0$, then $aaab, aaba, abaa, baaa$ are $4 \times 9 \times 8 = 288$ and $aabb, abab, abba$ are $3 \times 9 \times 8 = 216$ and $aaaa$ are 9

$$\therefore \text{Required probability} = \frac{288 + 216 + 9}{63 + 288 + 216 + 9} = \frac{513}{576} = \frac{57}{64}$$

6. (d) : Let E be the event that B throws a number more than 8. Then $P(E) = 1 - P(\bar{E})$.

$|\bar{E}|$ = Number of positive integral solutions of $x + y + z \leq 8$

$$\therefore |\bar{E}| = {}^8C_3 = 56 \text{ and } |S| = 216 \therefore P(\bar{E}) = \frac{56}{216} = \frac{7}{27}$$

7. (d) : With 10 dice, the number on each face being odd, we can never get an odd number as their sum.

8. (a) : Number of triplets $(3^r, 3^{r+1}, 3^{r+2})$
 $(0 \leq r \leq n)$ is $n - 1$
 Number of triplets $(3^r, 3^{r+2}, 3^{r+4})$ $(0 \leq r \leq n)$ is $n - 3$

$$\text{Number of triplets} \left(3^r, 3^{r+\frac{n-1}{2}}, 3^{r+n-1} \right) (n \text{ odd}) \text{ is } 2$$

$$\text{And number of triplets} \left(3^r, 3^{r+\frac{n}{2}}, 3^{r+n} \right) (n \text{ even}) \text{ is } 1$$

$\therefore$ If n is odd, the number of favourable outcomes

$$= (n-1) + (n-3) + \dots + 4 + 2 = \frac{n^2 - 1}{4}$$

And if n is even, the number of favourable outcomes

$$= (n-1) + (n-3) + \dots + 3 + 1 = \frac{n}{2} \times \frac{n}{2} = \frac{n^2}{4}$$

$$\frac{(n^2 - 1)}{(n+1)C_3} = \frac{3}{2n}, \text{ if } n \text{ is odd}$$

∴ Required probability = $\frac{4}{(n+1)C_3} = \frac{3}{2n}$, if n is odd

$$\frac{n^2}{(n+1)C_3} = \frac{4}{2(n^2 - 1)}, \text{ if } n \text{ is even}$$

9. (a) : We have, $f(x) = x^3 + ax^2 + bx + c$

∴ $f'(x) = 3x^2 + 2ax + b$. Now, $f(x)$ is increasing so,

$$f'(x) \geq 0 \quad \forall x$$

$$\Rightarrow 3x^2 + 2ax + b \geq 0 \Rightarrow D \leq 0$$

$$\therefore a^2 - 3b \leq 0$$

This is true for exactly 16 ordered pairs (a, b) where $1 \leq a, b \leq 6$, $(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)$ and $(4, 6)$.

$$\therefore \text{Required probability} = \frac{16}{36} = \frac{4}{9}$$

10. (c) : From 10 shoes, 6 can be selected in ${}^{10}C_6$ ways and if there is to be one pair among them it can be selected in ${}^5C_1$ ways. Other 4 shoes can be selected in ${}^4C_4(2^4) = 16$ ways.

$$\text{Hence, required probability} = \frac{5 \times 16}{{}^{10}C_6} = \frac{8}{21}$$

11. (d) : The sequence of consecutive heads may start with 1st toss or 2nd toss or 3rd toss ---- or at 6th toss. In any case, if it starts with r^{th} throw, the first $(r-2)$ throws may be head or tail but $(r-1)^{\text{th}}$ throw must be tail, after which again a head or tail can show up:

$$\underbrace{\text{HHH} \dots \text{H}}_{7 \text{ times}} \underbrace{\text{H X X X X X}}_{(r-2)} \underbrace{\text{X X X} \dots \text{X}}_{(r-1)^{\text{th}}} \underbrace{\text{T}}_{(r-1)^{\text{th}}} \underbrace{\text{HHH} \dots \text{H}}_{7 \text{ times}}$$

$$\therefore \text{Probability} = \frac{1}{2^7} + \frac{1}{2^7} + \frac{1}{2^7} + \dots + \frac{1}{2^7}$$

$$\frac{1}{2^7} \left[1 + \frac{5}{2} \right] = \frac{7}{2^8}$$

12. (a) : x and y can be any one of $(2n+1)$ numbers given.

Now, $|x - y| \leq n \Rightarrow x - n \leq y \leq x + n$

Hence number of possibilities of y , for $x = 0, 1, 2, 3, \dots, n-1, n, n+1, \dots, 2n$ are $n+1, n+2, n+3, \dots, 2n, 2n+1, 2n, 2n-1, \dots, n+1$ respectively.

$$\therefore \text{Probability} = \frac{2[(n+1) + (n+2) + \dots + (2n)] + 2n+1}{(2n+1)^2}$$

$$= \frac{3n^2 + 3n+1}{(2n+1)^2}$$

13. (d) : Total cases are when numbers ending with 3, 5, 7 or 8

Favourable cases are when numbers are ending with 3, 7, or 8

$$\text{So, the required probability} = \frac{3}{4}$$

14. (d) : Let the numbers be x, y . Given $x + y = 30$, $x, y \in N \Rightarrow x = 1, 2, 3, \dots, 29$

$$P(xy < 150) = P((30-x)x < 150) = P(x > 15 + \sqrt{75})$$

$$= P(x = 24, 25, 26, 27, 28, 29) = \frac{6}{29}$$

15. (a) : There are $\binom{n}{k}$ ways in which exactly k tails appear and in this case difference is $n - 2k$. Hence the expected value of $|H - T|$ is

$$\frac{1}{2^n} \sum_{k=0}^n |n-2k| \binom{n}{k} = \frac{1}{2^{n-1}} \sum_{k=0}^{\left[\frac{n}{2}\right]} (n-2k) \binom{n}{k} = \frac{n}{2^{n-1}} \left[\binom{n-1}{\frac{n}{2}} \right]$$

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MATH archives



Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE Main & Advanced Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE Main & Advanced. In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. The line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + 1 = z$ and $x + 1 = 2y = 2z$. The coordinates of each of the points of intersection are given by

- (a) (3, 3, 3) (1, 1, 1) (b) (3, 2, 3) (1, 1, 1)
(c) (3, 2, 3) (1, 1, 2) (d) (2, 3, 3) (2, 1, 1)

2. Total number of common tangents of $y^2 = 4ax$ and $xy = c^2$, is equal to

- (a) 1 (b) 2 (c) 3 (d) 4

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = [x]^2 + [x] - 2$, where $[\cdot]$ denotes greatest integer function, then $f(x)$ is

- (a) many-one into (b) many-one onto
(c) one-one into (d) one-one onto

4. If $2^{3n} - \alpha n - \beta$, $n \in \mathbb{N}$ is divisible by 49, then (α, β) is

- (a) $(-7, -1)$ (b) $(7, 1)$
(c) $(49, 1)$ (d) $(49, 7)$

5. The length of projection of the line segment joining the points $(1, -1, 0)$ and $(-1, 0, 1)$ to the plane $2x + y + 6z = 1$ is equal to

- (a) $\sqrt{\frac{255}{41}}$ (b) $\sqrt{\frac{237}{41}}$ (c) $\sqrt{\frac{137}{41}}$ (d) $\sqrt{\frac{155}{41}}$

6. $\sum_{r=1}^{15} \frac{r2^r}{(r+2)!}$ is equal to

- (a) $\frac{17! - 2^{16}}{17!}$ (b) $\frac{18! - 2^{17}}{18!}$

- (c) $\frac{16! - 2^{15}}{16!}$ (d) $\frac{15! - 2^{14}}{15!}$

7. There are n locks and n matching keys. If all the locks and keys are to be perfectly matched, then maximum number of trials is equal to

- (a) ${}^nC_2$ (b) $n - 1C_2$
(c) $n + 1C_2$ (d) none of these

8. Maximum length of the chord of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b \text{ such that eccentric angles of its}$$

extremities differ by $\frac{\pi}{2}$ is

- (a) $a\sqrt{2}$ (b) $b\sqrt{2}$
(c) $ab\sqrt{2}$ (d) none of these

9. The number of ordered pairs (m, n) ($m, n \in \{1, 2, \dots, 20\}$) such that $3^m + 7^n$ is a multiple of 10, is

- (a) 100 (b) 200
(c) $4! \times 4!$ (d) none of these

10. If the length of the tangent drawn from a variable point to one given circle is k ($k \neq 1$) times the length of the tangent from it to another circle, then the locus of the variable point is

- (a) an ellipse (b) a parabola
(c) a circle (d) a hyperbola

By : Prof. Shyam Bhushan, Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

SOLUTIONS

1. (b): The first line $x = y + 1 = z = \mu$ gives general point as $(\mu, \mu - 1, \mu)$. Second line $x + 1 = 2y = 2z = \lambda$ gives general point as $(\lambda - 1, \lambda/2, \lambda/2)$. The ratio of direction ratios of line joining points of intersection and direction cosines proportional to 2, 1, 2 are same

$$\text{i.e., } \frac{\mu - \lambda + 1}{2} = \frac{\mu - 1 - \frac{\lambda}{2}}{1} = \frac{\mu - \frac{\lambda}{2}}{2}$$

$$\Rightarrow \mu - \lambda + 1 = 2\mu - 2 - \lambda = \mu - \frac{\lambda}{2}$$

Solving, we get $\mu = 3, \lambda = 2$

Coordinates are $(3, 2, 3)$ $(1, 1, 1)$

2. (a): Any tangent of $xy = c^2$ is $\frac{x}{t} + \frac{y}{t} = 2c$

$$\text{i.e., } y = -\frac{x}{t^2} + \frac{2c}{t}$$

Comparing it with $y = mx + \frac{a}{m}$, we get

$$\frac{1}{1} = \frac{mt^2}{-1} = \frac{at}{2cm}$$

$$\Rightarrow m = -\frac{1}{t^2}, m = \frac{at}{2c} \Rightarrow \frac{at}{2c} = -\frac{1}{t^2} \Rightarrow t = -\left(\frac{2c}{a}\right)^{1/3}$$

So, there is only one common tangent.

3. (a): $f(x) = [x]^2 + [x] - 2 = ([x] + 2)([x] - 1)$

Now, $f(x) = 0$ for $x \in (1, 2)$ and $x \in [-2, -1)$

So, $f(x)$ is many one.

As $f(x)$ will take only integral values, so it is into.

4. (b): $8^n - \alpha n - \beta = (1 + 7)^n - \alpha n - \beta$
 $= 1 + 7n + {}^nC_2 7^2 + {}^nC_3 7^3 + \dots + {}^nC_n 7^n - \alpha n - \beta$
 $= (1 - \beta) + (7 - \alpha)n + 49({}^nC_2 + {}^nC_3 7 + \dots + {}^nC_n 7^{n-2})$

$\therefore$ If $8^n - \alpha n - \beta$ is divisible by 49, then
 $1 - \beta = 0, 7 - \alpha = 0 \Rightarrow \beta = 1, \alpha = 7$.

5. (b): $A \equiv (1, -1, 0), B \equiv (-1, 0, 1)$

Direction ratios of segment AB are 2, -1, -1.

If θ be the acute angle between segment AB and normal

$$\text{to plane, then } \cos \theta = \frac{|2 \times 2 - 1 \times 1 - 1 \times 6|}{\sqrt{4+1+36} \sqrt{4+1+1}} = \frac{3}{\sqrt{246}}$$

Length of projection = $AB \sin \theta$

$$= \sqrt{6} \sqrt{1 - \frac{9}{246}} = \sqrt{\frac{237}{41}} \text{ units}$$

6. (a): We have, $\frac{r2^r}{(r+2)!} = \frac{(r+2-2)2^r}{(r+2)!}$

$$= \frac{2^r}{(r+1)!} - \frac{2^{r+1}}{(r+2)!}$$

$$\therefore \sum_{r=1}^{15} \frac{r2^r}{(r+2)!} = \left(\frac{2}{2!}\right) - \left(\frac{2^{16}}{17!}\right) = 1 - \frac{2^{16}}{17!}$$

7. (c): For the first key, maximum number of trials needed is n . For second key, it will be $(n-1)$.

In general, for r^{th} key, maximum number of trials needed is $(n-r+1)$, then total number of trials needed

$$= n + (n-1) + (n-2) + \dots + 1 = \frac{n(n+1)}{2} = {}^{n+1}C_2$$

8. (a): Let the extremities of the chord be

$$P_1 \equiv (a \cos \theta, b \sin \theta) \text{ and } P_2 \equiv (-a \sin \theta, b \cos \theta)$$

$$\text{Now } (P_1 P_2)^2 = a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$\Rightarrow (P_1 P_2)^2 = a^2 + b^2 + (a^2 - b^2) \sin 2\theta \leq a^2 + b^2 + a^2 - b^2$$

$$\Rightarrow (P_1 P_2)^2 \leq 2a^2 \Rightarrow P_1 P_2 \leq a\sqrt{2}$$

9. (a): The last digit of any power of 3 can be 3, 9, 7, 1.

Similarly last digit of any power of 7 can be 7, 9, 3, 1

Total number of ways = $5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 = 100$.

10. (c): We have $\sqrt{S_1} = k\sqrt{S_2} \Rightarrow S_1 = k^2 S_2$

So, locus is $x^2 + y^2 + 2g_1 x + 2f_1 y + c_1$

$$= k^2(x^2 + y^2 + 2g_2 x + 2f_2 y + c_2)$$

$$\Rightarrow (x^2 + y^2)(1 - k^2) + 2x(g_1 - k^2 g_2) + 2y(f_1 - k^2 f_2)$$

$$+ c_1 - k^2 c_2 = 0, \text{ which is a circle.}$$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6×6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

11+	2+		20×	6×	
	3-			3+	
240×		6×			
		6×	7+	30×	
6×					9+
8+			2+		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

OLYMPIAD CORNER



1. Prove that the average of the numbers $n \sin n^\circ$, $n = 2, 4, 6, \dots, 180$ is $\cot 1^\circ$.
2. The inscribed circumference in the triangle ABC is tangent to BC , CA and AB at D , E and F , respectively. Suppose that this circumference meets AD again at its mid-point X ; that is, $AX = XD$. The lines XB and XC meet the inscribed circumference again at Y and Z , respectively. Show that $EY = FZ$.
3. There are 1994 employees in the office. Each of them knows 1600 others of them. Prove that we can find 6 employees, each of them knowing all 5 others.
4. A triangle ABC is given. From the vertex B , n rays are constructed intersecting the side AC . For each of the $n+1$ triangles obtained, an incircle with radius r_i and excircle (which touches the side AC) with radius R_i is constructed. Prove that the expression $\frac{r_1 r_2 \dots r_{n+1}}{R_1 R_2 \dots R_{n+1}}$ depends on neither n nor on which rays are constructed.
5. Let $ABCD$ be an inscribed quadrilateral. Its diagonals intersect at O . Let the midpoints of AB and CD be U and V . Prove that the lines through O , U and V , perpendicular to AD , BD and AC respectively, are concurrent.
6. Triangles BCD and ACE are constructed outwardly on sides BC and CA of triangle ABC such that $AE = BD$ and $\angle BDC + \angle AEC = 180^\circ$. The point F is chosen to lie on the segment AB so that $\frac{AF}{FB} = \frac{DC}{CE}$.
Prove that $\frac{DE}{CD + CE} = \frac{EF}{BC} = \frac{FD}{AC}$.
7. Let A , B and C be non-collinear points. Prove that there is a unique point X in the plane of ABC such

$$\text{that } XA^2 + XB^2 + AB^2 = XB^2 + XC^2 + BC^2 = XC^2 + XA^2 + CA^2.$$

SOLUTIONS

1. Let $x = 2\sin 2^\circ + 4\sin 4^\circ + \dots + 90\sin 90^\circ + \dots + 178\sin 178^\circ$
 $= (2 + 178)\sin 2^\circ + (4 + 176)\sin 4^\circ + \dots + 90\sin 90^\circ$
 $= 180(\sin 2^\circ + \sin 4^\circ + \dots + \sin 88^\circ) + 90\sin 90^\circ$.
 Then, $\bar{x} = \frac{x}{90} = 2\sin 2^\circ + 2\sin 4^\circ + \dots + 2\sin 88^\circ + 1$
 $\bar{x}\sin 1^\circ = 2\sin 2^\circ \sin 1^\circ + 2\sin 4^\circ \sin 1^\circ + \dots + 2\sin 88^\circ \sin 1^\circ + \sin 1^\circ$.
 Now, $2\sin 2^\circ \sin 1^\circ = \cos 1^\circ - \cos 3^\circ$
 $2\sin 4^\circ \sin 1^\circ = \cos 3^\circ - \cos 5^\circ$
 $2\sin 88^\circ \sin 1^\circ = \cos 87^\circ - \cos 89^\circ$.
 Hence, $\bar{x}\sin 1^\circ = \cos 1^\circ - \cos 89^\circ + \sin 1^\circ = \cos 1^\circ$

Thus $\bar{x} = \cot 1^\circ$, as required.

2. Since $\angle BFY = \angle BXF$ and $\angle FBY = \angle XBF$ we have $\angle BFY$ and $\angle BXF$ are similar, so that $FY : FX = BF : BX$ (1)
 Similarly, we get $DY : DX = BD : BX$ (2)

As $BF = BD$, we have from

(1) and (2) that

$$FY : FX = DY : DX.$$

Since $AX = DX$ we get

$$FY : FX = DY : AX. \dots (3)$$

Since X , F , Y , D are

concylic, we have

$$\angle FYD = \angle AXF. \dots (4)$$

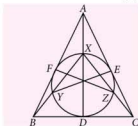
Thus, we get from (3) and

(4) that $\triangle FYD$ is similar to $\triangle AXF$.

Hence, $\angle YFD = \angle XFA = \angle XDF$ so that $FY \parallel XD$.

Similarly we have $EZ \parallel XD$. Thus $FY \parallel EZ$.

Therefore, $FYZE$ is an isosceles trapezoid and then $EY = FZ$.



3. Let E denote the set of these 1994 employees. For each $x \in E$, let $S(x)$ denote the set of all employees whom x does not know. Then by assumption, $|S(x)| = 393$ for all $x \in E$. Let a and b be any two employees who know each other.

Since $|S(a) \cup S(b)| \leq 2 \times 393 = 786 < 1992$, $\exists c \in E$ such that a, b and c form a triple of mutual acquaintances. Since $|S(a) \cup S(b) \cup S(c)| \leq 3 \times 393 = 1179 < 1991$, $\exists d \in E$ such that a, b, c and d form a quadruple of mutual acquaintances.

Since $|S(a) \cup S(b) \cup S(c) \cup S(d)| \leq 4 \times 393 = 1572 < 1990$, $\exists e \in E$ such that a, b, c, d and e form a quintuple of mutual acquaintances finally,

Since $|S(a) \cup S(b) \cup S(c) \cup S(d) \cup S(e)| \leq 5 \times 393 = 1965 < 1989$,

$\exists f \in E$ such that a, b, c, d, e and f form a sextuple of mutual acquaintances.

4. If A, B, C are the angles of a triangle, then

$$r = \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \quad \text{and} \quad r_x = \tan \frac{A}{2},$$

where r, r_x are the inradius and the radius of the excircle opposite A , and s is the semiperimeter.

It follows that

$$\frac{r}{r_x} = \tan \frac{B}{2} \tan \frac{C}{2}.$$

Next we apply this result to each of $n+1$ triangles obtained. This yields,

$$\frac{r_1}{R_1} = \tan \frac{A}{2} \tan \frac{\alpha_1}{2}$$

$$\frac{r_2}{R_2} = \tan \frac{180^\circ - \alpha_1}{2} \tan \frac{\alpha_2}{2}$$

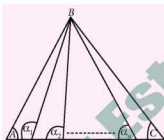
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$$\frac{r_n}{R_n} = \tan \frac{180^\circ - \alpha_{n-1}}{2} \tan \frac{\alpha_n}{2}$$

$$\frac{r_{n+1}}{R_{n+1}} = \tan \frac{180^\circ - \alpha_n}{2} \tan \frac{C}{2}.$$

Multiplying these equalities, we observe that the product of all the right hand members is

$$\begin{aligned} \tan \frac{A}{2} \cdot \left(\tan \frac{\alpha_1}{2} \cot \frac{\alpha_1}{2} \right) \cdot \left(\tan \frac{\alpha_2}{2} \cot \frac{\alpha_2}{2} \right) \\ \dots \left(\tan \frac{\alpha_n}{2} \cot \frac{\alpha_n}{2} \right) \cdot \tan \frac{C}{2} = \tan \frac{A}{2} \cdot \tan \frac{C}{2}, \end{aligned}$$

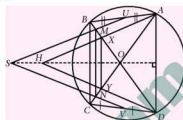


and we get

$$\frac{r_1 r_2 \dots r_{n+1}}{R_1 R_2 \dots R_{n+1}} = \tan \frac{A}{2} \cdot \tan \frac{C}{2}$$

which depends neither on n nor on which rays are constructed.

5. Case 1. Neither is AC orthogonal to BD , nor is AD a diameter.



Let M, N be the feet of the perpendiculars from U, V to BD, AC respectively, and let S be the intersection of UM and VN . Let X, Y be the feet of the perpendiculars from A, D to BD, AC respectively, and let H be the intersection of AX and DY . Since U is the midpoint of AB , and $UM \parallel AX$, M is the midpoint of BX . Similarly, N is the midpoint of CY .

SAMURAI SUDOKU

SOLUTION · OCTOBER 2018



6	5	1	4	9	8	3	2	7
2	4	3	6	7	1	5	9	8
8	9	7	2	5	3	6	1	4
4	6	9	1	2	7	8	3	5
7	3	2	8	4	5	9	6	1
1	8	5	9	3	6	4	7	2
5	2	4	3	1	9	7	8	6
3	7	8	5	6	2	1	4	9
9	1	6	7	8	4	2	5	3
6	3	7	4	2	9	5	1	8
4	2	1	5	7	8	6	3	9
5	9	8	6	1	3	7	4	2
1	3	8	2	5	6	9	7	4
6	2	7	9	3	4	8	1	5
9	5	4	7	1	8	3	6	2
5	6	3	4	7	2	1	8	9
8	4	2	1	9	3	7	5	6
7	1	9	8	6	5	2	4	3
4	7	6	3	2	1	5	9	8
3	9	5	6	8	7	4	2	1
2	8	1	5	4	9	6	3	7
4	2	3	1	5	7	9	6	8
9	7	1	5	3	4	2	6	8
6	1	9	8	2	5	7	3	4
5	8	4	3	7	6	9	2	1
7	3	2	1	4	9	5	8	6
1	6	5	7	8	2	3	4	9
2	4	7	6	9	3	8	1	5
3	9	8	4	5	1	6	7	2

Since $\angle AXD = \angle AYD = 90^\circ$, A, X, Y, D are concyclic. Therefore $\angle YXD = \angle YAD = \angle CAD = \angle CBD$. Thus we have $XY \parallel BC$. Since, M, N are the midpoints of BX, CY respectively, we have $MN \parallel XY$.

Since, $SM \parallel HX, XN \parallel HY$, and $MN \parallel XY, MX, NY$ and SH are concurrent at O . Therefore, S, H, O are collinear.

Since $AH \perp OD$ and $DH \perp OA$, H is the orthocentre of $\triangle OAD$, so that $HO \perp AD$. Thus we have $SO \perp AD$.

Thus, the lines through O, U and V , perpendicular to AD, BD and AC respectively, are concurrent at S .

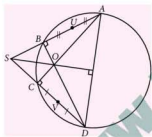
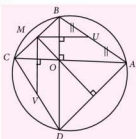
Case 2. AC is orthogonal to BD .

Let M be the midpoint of BC . Since U is the midpoint of AB , we get $UM \perp AC$, so that $UM \perp BD$. Similarly we have $VM \perp AC$.

Since, $AC \perp BD$ and M is the midpoint of BC , by Brahmagupta's Theorem we have $MO \perp AD$.

Thus the lines through O, U and V , perpendicular to AD, BD and AC respectively, are concurrent at M .

Case 3. AD is a diameter.



Let S be the intersection of AB and CD . Since $AB \perp BD$ and $CD \perp AC$, S is the orthocentre of $\triangle OAD$. Thus $SO \perp AD$. Hence, the lines through O, U and V , perpendicular to AD, BD and AC respectively, are concurrent at S .

6. Let G be a point on AE produced beyond E such that $\angle ECG = \angle DCB$. Since $\angle CEG = 180^\circ - \angle AEC = \angle CDB$, we have $\triangle CEG \sim \triangle CDB$, (directly similar) from which we have $\triangle CBG \sim \triangle CDE$. Thus

$$\angle BCG = \angle DEC. \quad \dots(1)$$

$$\text{Since } \frac{AF}{FB} = \frac{CD}{CE} = \frac{BD}{EG} = \frac{AE}{EG}, \text{ we get } FE \parallel BG, \text{ so that}$$

$$\angle AGB = \angle AEF. \quad \dots(2)$$

Hence we have from (1) and (2)

$$\begin{aligned} \angle FED &= \angle AEC - (\angle AEF + \angle DEC) \\ &= \angle AEC - (\angle AGB + \angle BGC) \\ &= \angle AEC - \angle AGC \\ &= \angle ECG \\ &= \angle BCD. \end{aligned} \quad \dots(3)$$

Similarly, we have

$$\angle FDE = \angle ACE. \quad \dots(4)$$

Let H be a point on CD produced beyond D such that $DH = EC$. Since $BD = AE$ and $\angle BDH = 180^\circ - \angle BDC = \angle AEC$, we have $\triangle BDH \cong \triangle AEC$,

so that $BH = AC$, and $\angle BHD = \angle ACE$.

As $\angle FED = \angle BCD = \angle BCH$, and $\angle FDE = \angle ACE = \angle BHD = \angle BHC$, we have $\triangle FDE \sim \triangle BHC$.

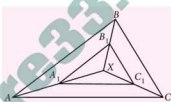
Thus we get

$$\frac{DE}{HC} = \frac{EF}{BC} = \frac{FD}{BH}.$$

Since, $HC = CD + DH = CD + CE$, and $BH = AC$, we have

$$\frac{DE}{CD + CE} = \frac{EF}{BC} = \frac{FD}{AC}.$$

7.



From the hypothesis we have

$$AX^2 + AB^2 = CX^2 + CB^2. \quad \dots(1)$$

If B_1 is the midpoint of BC , applying the first theorem of the median in the triangle $\triangle ABX, \triangle CBX$ we get

$$2AB_1^2 + 2BB_1^2 = 2CB_1^2 + 2BB_1^2 \quad \dots(2)$$

or $AB_1 = CB_1$. This indicates that the perpendicular bisector of the side AC passes through the point B_1 . Let A_1, C_1 be the midpoints of AX and CX , respectively.

Similarly, we obtain that the perpendicular bisectors of BC and AB pass through the midpoints A_1 and C_1 , respectively.

Furthermore we obtain $AB \parallel A_1B_1, AC \parallel A_1C_1$ and $BC \parallel B_1C_1$.

From (3) and (4) we get that the circumcentre O of $\triangle ABC$ is the orthocentre H_1 of $\triangle A_1B_1C_1$.

Also from (4) the triangles ABC and $A_1B_1C_1$ are similar with X the centre of similarity and ratio $1/2$.

So, their orthocentres H and H_1 lie in the same straight line with the point X and $HH_1 = H_1X$.

Combining (5) and (7) we get $HO = OX$; that is the point X is known (constant), because X is symmetric to H with respect to the orthocentre O of $\triangle ABC$.



MODEL TEST PAPER 2019

JEE MAIN

JEE Main I between
6th to 20th January
and

JEE Main II between
6th to 20th April 2019

1. Total number of values of 'a' so that $x^2 - x - a = 0$ has integral roots, where $a \in \mathbb{N}$ and $6 \leq a \leq 100$, is equal to

- (a) 2 (b) 4 (c) 6 (d) 8

2. If α, β, γ are the roots of $x^3 - x^2 - 1 = 0$, then the value of $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ is equal to

- (a) -5 (b) -6 (c) -7 (d) -2

3. $\angle A, \angle B, \angle C$ of a triangle ABC are in A.P. If a, b, c are the corresponding sides, then

- (a) $b^2 + c^2 - bc = a^2$ (b) $a^2 + c^2 - ac = b^2$
(c) $b^2 + a^2 - ab = c^2$ (d) none of these

4. Total number of positive real values of x such that $x, [x], \{x\}$ are in H.P., where $[\cdot]$ denotes the greatest integer function and $\{\cdot\}$ denotes fractional part, is equal to

- (a) zero (b) 1
(c) 2 (d) none of these

5. $1, z_1, z_2, z_3, \dots, z_{n-1}$ are n^{th} roots of unity, then the value of $\frac{1}{3-z_1} + \frac{1}{3-z_2} + \dots + \frac{1}{3-z_{n-1}}$ is equal to

- (a) $\frac{n3^{n-1}}{3^n - 1} + \frac{1}{2}$ (b) $\frac{n3^{n-1}}{3^n - 1} - 1$
(c) $\frac{n3^{n-1}}{3^n - 1} + 1$ (d) none of these

6. If $\left| \frac{z_1 - 2z_2}{2 - z_1 \bar{z}_2} \right| = 1$ and $|z_2| \neq 1$, then value of $|z_1|$ is

- equal to
(a) 2 (b) 1
(c) 4 (d) none of these

7. $\sum_{r=0}^n \frac{{}^nC_r}{r+2}$ is equal to

- (a) $\frac{n \cdot 2^{n+1}}{(n+1)(n+2)}$ (b) $\frac{n \cdot 2^{n+2}}{(n+1)(n+2)}$

- (c) $\frac{n \cdot 2^n}{(n+1)(n+2)}$ (d) none of these

8. Total number of positive integral solutions of the equation $x_1 \cdot x_2 \cdot x_3 = 60$ is equal to

- (a) 27 (b) 54
(c) 64 (d) none of these

9. Let $\vec{a}_r = x_r \hat{i} + y_r \hat{j} + z_r \hat{k}$, $r = 1, 2, 3$, be three mutually perpendicular unit vectors, then the value of $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ is equal to

- (a) zero (b) ± 1
(c) ± 2 (d) none of these

10. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other, then $[\vec{a} + (\vec{a} \times \vec{b}), \vec{b} + (\vec{a} \times \vec{b}), \vec{a} \times \vec{b}]$ will always be equal to

- (a) 1 (b) zero
(c) -1 (d) none of these

11. Let $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that is equally inclined to $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ is

- (a) $\frac{1}{\sqrt{2}}(\vec{a} + \vec{b} + \vec{a} \times \vec{b})$ (b) $\frac{1}{2}(\vec{a} \times \vec{b} + \vec{a} + \vec{b})$
(c) $\frac{1}{\sqrt{3}}(\vec{a} + \vec{b} + \vec{a} \times \vec{b})$ (d) $\frac{1}{3}(\vec{a} \times \vec{b} + \vec{a} + \vec{b})$

12. Two fair and ordinary dice are rolled simultaneously. The probability that neither the sum of outcomes is 9 nor the dice show equal outcomes, is equal to

- (a) 8/9 (b) 1/9 (c) 15/18 (d) 13/18

13. Three integers are selected simultaneously from the set of integers $\{1, 2, 3, 4, \dots, 50\}$. The probability that the selected numbers are consecutive, is equal to

- (a) $\frac{9}{(25)(49)}$ (b) $\frac{6}{(25)(49)}$
 (c) $\frac{3}{(25)(49)}$ (d) none of these

14. If $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$, then ordered triplet

(x, y, z) is

- (a) $\left(-\frac{9}{4}, 2, \frac{7}{4}\right)$ (b) $\left(\frac{7}{4}, 2, -\frac{9}{4}\right)$
 (c) $\left(\frac{9}{4}, -2, \frac{7}{4}\right)$ (d) $\left(\frac{7}{4}, -2, \frac{9}{4}\right)$

15. $f: N \rightarrow N$ where $f(x) = x - (-1)^x$, then f is

- (a) one one and into (b) many one and into
 (c) one one and onto (d) many one and onto

16. Let $f: [0, \infty) \rightarrow [0, \infty)$ and $g: [0, \infty) \rightarrow [0, \infty)$ be non-increasing and non-decreasing functions respectively, $h(x) = g(f(x))$. If f and g are differentiable for all points in their respective domains and $h(0) = 0$, then $h(x)$ will always be

- (a) an increasing function
 (b) a decreasing function
 (c) identically zero
 (d) none of these

17. If $\int \frac{x^3 dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + c$, then

- (a) $a = \frac{1}{3}, b = 1$ (b) $a = -\frac{1}{3}, b = 1$
 (c) $a = -\frac{1}{3}, b = -1$ (d) $a = \frac{1}{3}, b = -1$

18. Let $f(x) = \begin{cases} x|x| & , x \leq -1 \\ [x+1] + [1-x] & , -1 < x < 1 \\ -x|x| & , x \geq 1 \end{cases}$, where

$[.]$ denotes the greatest integer function. Then the value

of $\int_{-2}^2 f(x) dx$, is equal to

- (a) $-\frac{8}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

19. Area bounded by the curves $y = \sin x$, tangent drawn to it at $x = 0$ and the line $x = \frac{\pi}{2}$, is equal to

- (a) $\frac{\pi^2 - 4}{2}$ sq. units (b) $\frac{\pi^2 - 8}{8}$ sq. units
 (c) $\frac{\pi^2 - 2}{4}$ sq. units (d) $\frac{\pi^2 - 2}{2}$ sq. units

20. The differential equation of the family of circles, having centres on the x -axis is

- (a) $y \cdot \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - 1 = 0$
 (b) $y \cdot \frac{d^2 y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 1 = 0$
 (c) $y \cdot \frac{d^2 y}{dx^2} - \left(\frac{dy}{dx}\right)^2 - 1 = 0$
 (d) $y \cdot \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$

21. A variable line is drawn through the intersecting point of the lines $3x + 4y - 12 = 0$ and $x + 2y - 5 = 0$, meeting the co-ordinate axes at the points A and B . Locus of midpoint of segment AB is

- (a) $4x + 3y = 4xy$ (b) $3x + 4y = 3xy$
 (c) $3x + 4y = 4xy$ (d) none of these

22. If chord $x \cos \theta + y \sin \theta = p$ of $x^2 + y^2 = a^2$ subtends a right angle at the origin, then

- (a) $a^2 = p^2$ (b) $a^2 = 2p^2$
 (c) $a^2 = 3p^2$ (d) none of these

23. Locus of the midpoint of any focal chord of $y^2 = 4ax$ is

- (a) $y^2 = a(x - 2a)$ (b) $y^2 = 2a(x - 2a)$
 (c) $y^2 = 2a(x - a)$ (d) none of these

24. The equation of common tangents of the curves $x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

- (a) $y - 2x - 4 = 0, y + 2x + 4 = 0$
 (b) $y - 2x - 2 = 0, y + 2x + 2 = 0$
 (c) $2y - x - 4 = 0, 2y + x + 4 = 0$
 (d) $2y - x - 2 = 0, 2y + x + 2 = 0$

25. Length of perpendicular from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent of the ellipse that makes equal angles with coordinate axes is

- (a) $\sqrt{\frac{a^2 - b^2}{2}}$ (b) $\sqrt{\frac{a^2 + b^2}{2}}$
 (c) $\frac{\sqrt{a^2 - b^2}}{2}$ (d) $\frac{\sqrt{a^2 + b^2}}{2}$

26. The distance of the point $(-1, 2, 6)$ from the line $\frac{x-2}{6} = \frac{y-3}{3} = \frac{z+4}{-4}$, is equal to

- (a) 7 units (b) 9 units
(c) 10 units (d) 12 units

27. If $\frac{\sin x}{\sin y} = \frac{1}{2}$, $\frac{\cos x}{\cos y} = \frac{3}{2}$ where $x, y \in \left(0, \frac{\pi}{2}\right)$, then

the value of $\tan(x+y)$ is equal to

- (a) $\sqrt{13}$ (b) $\sqrt{14}$ (c) $\sqrt{17}$ (d) $\sqrt{15}$

28. If $\{\cot^{-1}x\} + \{\cos^{-1}x\} = 0$, where $\{.\}$ denotes the greatest integer function, then complete set of values of 'x' is

- (a) $(\cos 1, 1]$ (b) $(\cos 1, \cot 1)$
(c) $(\cot 1, 1)$ (d) none of these

29. In any triangle ABC , the value of $1 - \tan \frac{B}{2} \cdot \tan \frac{C}{2}$ is equal to

- (a) $\frac{2a}{a+b+c}$ (b) $\frac{2b}{a+b+c}$
(c) $\frac{2c}{a+b+c}$ (d) none of these

30. Domain of $f(x) = \ln(ax^3 + (a+b)x^2 + (b+c)x + c)$, where $a > 0$, $b^2 - 4ac = 0$, is

- (a) $(-1, \infty) \sim \left\{-\frac{b}{2a}\right\}$ (b) $(1, \infty) \sim \left\{-\frac{b}{2a}\right\}$
(c) $(-1, 1) \sim \left\{-\frac{b}{2a}\right\}$ (d) none of these

SOLUTIONS

1. (d): Here, $x^2 - x - a = 0 \therefore D = 1 + 4a$ (odd)

Let $D = (2\lambda + 1)^2$

$$\Rightarrow 1 + 4a = 1 + 4\lambda^2 + 4\lambda \Rightarrow a = \lambda(\lambda + 1)$$

Since $a \in [6, 100]$

$$\therefore a = 6, 12, 20, 30, 42, 56, 72, 90$$

Thus a can attain 8 different values.

2. (a): We have, $x^3 - x^2 - 1 = 0$

$$\therefore \Sigma \alpha = 1, \Sigma \alpha\beta = 0, \alpha\beta\gamma = 1$$

$$\Sigma \frac{1+\alpha}{1-\alpha} = \Sigma \frac{2-(1-\alpha)}{1-\alpha} = \Sigma \left(\frac{2}{1-\alpha} - 1 \right) = 2 \Sigma \frac{1}{1-\alpha} - 3 \dots (i)$$

$$\text{Now, } \frac{1}{(x-\alpha)} + \frac{1}{(x-\beta)} + \frac{1}{(x-\gamma)} = \frac{3x^2 - 2x}{x^3 - x^2 - 1}$$

$$\Rightarrow \frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = \frac{3-2}{1-1-1} = -1$$

$$\Rightarrow \Sigma \frac{1+\alpha}{1-\alpha} = 2(-1) - 3 = -5 \quad [\text{Using (i)}]$$

3. (b): Given $\angle A, \angle B, \angle C$ of a triangle ABC are in A.P. $\therefore 2\angle B = \angle A + \angle C \Rightarrow \angle B = \frac{\pi}{3}$

$$\text{Now, } \cos B = \cos \frac{\pi}{3} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow a^2 + c^2 - ac = b^2$$

4. (b): Given $x, [x], \{x\}$ are in H.P.

$$\therefore [x] = \frac{2\{x\}x}{\{x\} + x}$$

$$\Rightarrow [x]\{x\} + [x](\{x\} + \{x\}) = 2\{x\}(\{x\} + \{x\})$$

$$\Rightarrow [x]^2 = 2\{x\}^2$$

$$\Rightarrow \{x\}^2 = \frac{[x]^2}{2} \Rightarrow 0 < \frac{[x]^2}{2} < 1$$

$$\Rightarrow 0 < [x]^2 < 2$$

$$\Rightarrow [x] = 1 \Rightarrow \{x\} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}}, \text{ is the only possible value of } x.$$

5. (d): Since $1, z_1, z_2, \dots, z_{n-1}$ are the n^{th} roots of unity So, $(z^n - 1) = (z - 1)(z - z_1)(z - z_2) \dots (z - z_{n-1})$

$$\Rightarrow \frac{nz^{n-1}}{(z^n - 1)} = \frac{1}{z-1} + \frac{1}{z-z_1} + \frac{1}{z-z_2} + \dots + \frac{1}{z-z_{n-1}}$$

Putting $z = 3$, we get

$$\frac{n \cdot 3^{n-1}}{3^n - 1} = \frac{1}{2} + \frac{1}{3-z_1} + \frac{1}{3-z_2} + \dots + \frac{1}{3-z_{n-1}}$$

$$\Rightarrow \frac{1}{3-z_1} + \frac{1}{3-z_2} + \dots + \frac{1}{3-z_{n-1}} = \frac{n \cdot 3^{n-1}}{3^n - 1} - \frac{1}{2}$$

$$6. (a): \text{ We have, } \left| \frac{z_1 - 2z_2}{2 - z_1\bar{z}_2} \right| = 1$$

$$\Rightarrow |z_1 - 2z_2| = |2 - z_1\bar{z}_2|$$

$$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$$

$$\Rightarrow |z_1 - 2z_2|(\bar{z}_1 - 2\bar{z}_2) = (2 - \bar{z}_1\bar{z}_2)(2 - \bar{z}_1z_2)$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 - 2z_1\bar{z}_2 - 2\bar{z}_1z_2$$

$$= 4 - 2\bar{z}_1z_2 - 2z_1\bar{z}_2 + |z_1|^2|z_2|^2$$

$$\Rightarrow |z_1|^2(1 - |z_2|^2) - 4(1 - |z_2|^2) = 0$$

$$\Rightarrow (1 - |z_2|^2)(|z_1|^2 - 4) = 0$$

$$\Rightarrow |z_1| = 2$$

$$[\because |z_2| \neq 1]$$

7. (d): Here, $(1+x)^n = \sum_{r=0}^n {}^nC_r x^r$

$$\Rightarrow x \cdot (1+x)^n = \sum_{r=0}^n {}^nC_r x^{r+1}$$

Integrating both sides within the limits 0 to 1, we get

$$\begin{aligned}\int_0^1 x(1+x)^n dx &= \sum_{r=0}^n \frac{n C_r}{r+2} \\&\Rightarrow \sum_{r=0}^n \frac{n C_r}{r+2} = \frac{x \cdot (1+x)^{n+1}}{n+1} \bigg|_0^1 - \int_0^1 \frac{(1+x)^{n+1}}{n+1} dx \\&= \frac{2^{n+1}}{n+1} - \frac{2^{n+2}-1}{(n+1)(n+2)} = \frac{n \cdot 2^{n+1} + 1}{(n+1)(n+2)}\end{aligned}$$

8. (b): We have, $x_1 \cdot x_2 \cdot x_3 = 2^2 \cdot 3 \cdot 5$

$\Rightarrow$ Total positive integral solutions = $3 \cdot 3 \cdot 4 C_2 = 54$

9. (b): Let $\Delta = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

Now, $\Delta^2 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

$$= \begin{vmatrix} x_1^2 + y_1^2 + z_1^2 & x_1 x_2 + y_1 y_2 + z_1 z_2 & x_1 x_3 + y_1 y_3 + z_1 z_3 \\ x_1 x_2 + y_1 y_2 + z_1 z_2 & x_2^2 + y_2^2 + z_2^2 & x_2 x_3 + y_2 y_3 + z_2 z_3 \\ x_1 x_3 + y_1 y_3 + z_1 z_3 & x_2 x_3 + y_2 y_3 + z_2 z_3 & x_3^2 + y_3^2 + z_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$\Rightarrow \Delta = \pm 1$

10. (a): We have, $[\vec{a} + (\vec{a} \times \vec{b}), \vec{b} + (\vec{a} \times \vec{b}), \vec{a} \times \vec{b}]$
 $= (\vec{a} + (\vec{a} \times \vec{b})) \cdot ((\vec{b} + (\vec{a} \times \vec{b})) \times (\vec{a} \times \vec{b}))$
 $= (\vec{a} + (\vec{a} \times \vec{b})) \cdot (\vec{b} \times (\vec{a} \times \vec{b}))$
 $= (\vec{a} + (\vec{a} \times \vec{b})) \cdot (\vec{a} - (\vec{a} \cdot \vec{b})\vec{b}) = \vec{a} \cdot \vec{a} + \vec{a} \cdot (\vec{a} \times \vec{b}) [\because \vec{a} \cdot \vec{b} = 0]$
 $= 1 [\because \vec{a} \cdot (\vec{a} \times \vec{b}) = 0]$

11. (c): Let the required unit vector $\vec{r}$ be

$$\vec{r} = x_1 \vec{a} + x_2 \vec{b} + x_3 \vec{a} \times \vec{b}$$

We must have $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot (\vec{a} \times \vec{b})$

(as $\vec{r}, \vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ all are unit vectors)

$$\therefore \vec{r} \cdot \vec{a} = x_1, \vec{r} \cdot \vec{b} = x_2, \vec{r} \cdot (\vec{a} \times \vec{b}) = x_3$$

$$\Rightarrow \vec{r} = \lambda(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$$

Also, $\vec{r} \cdot \vec{r} = 1$

$$\Rightarrow \lambda^2(\vec{a} + \vec{b} + (\vec{a} \times \vec{b})) \cdot (\vec{a} + \vec{b} + (\vec{a} \times \vec{b})) = 1$$

$$\Rightarrow \lambda^2(\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{b}) \cdot \vec{a} + (\vec{a} \times \vec{b}) \cdot \vec{b} + (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{b})) = 1$$

$$\Rightarrow \lambda^2 = \frac{1}{3} \Rightarrow \lambda = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm \frac{1}{\sqrt{3}}(\vec{a} + \vec{b} + \vec{a} \times \vec{b})$$

12. (d): Sum of outcomes can be equal to nine in four ways namely (3, 6), (6, 3), (4, 5), (5, 4). Equal outcomes can occur in 6 ways namely, (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).

Thus total favourable outcomes = $36 - (4 + 6) = 26$

$$\text{Thus, required probability} = \frac{26}{36} = \frac{13}{18}$$

13. (c): Possible set of three consecutive numbers are {1, 2, 3}, {2, 3, 4}, {3, 4, 5}, ..., {48, 49, 50}, which are 48 in number.

$$\text{Thus, required probability} = \frac{{}^{48}C_1}{{}^{50}C_3} = \frac{3}{(25)(49)}$$

14. (b): We have, $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x+2y+z \\ y \\ 3x-y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

$$\Rightarrow y = 2, -x + 2y + z = 0, 3x - y + z = 1$$

$$\Rightarrow x - z = 4, 3x + z = 3 \Rightarrow x = \frac{7}{4}, z = -\frac{9}{4}$$

15. (c): Here, $f(x) = \begin{cases} x-1, & \text{if } x = \text{even} \\ x+1, & \text{if } x = \text{odd} \end{cases}$

which is clearly one one and onto.

16. (c): Given, $h(x) = g(f(x))$ since $g(x) \in [0, \infty)$

$$\Rightarrow h(x) \geq 0 \quad \forall x \in \text{domain.}$$

$$\text{Also, } h'(x) = g'(f(x)) \cdot f'(x) \leq 0$$

$$\text{as } g'(x) \geq 0 \text{ and } f'(x) \leq 0$$

$$\Rightarrow h(x) \text{ is non-increasing.}$$

$$\text{Thus } h(x) \leq 0 \quad \forall x \in \text{domain as } h(0) = 0$$

$$\text{Hence } h(x) = 0 \quad \forall x \in \text{domain.}$$

17. (d): Here, $\int \frac{x^3 dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + c$

Differentiating both sides w.r.t. x , we get

$$\frac{x^3}{\sqrt{1+x^2}} = \frac{3a}{2}(1+x^2)^{1/2} \cdot 2x + \frac{bx}{\sqrt{1+x^2}}$$

$$\Rightarrow x^3 = 3x(1+x^2)a + bx$$

$$\Rightarrow b + 3a = 0, 3a = 1$$

$$\Rightarrow a = \frac{1}{3}, b = -1$$

18. (a) : Here, $f(x) = \begin{cases} -x^2, & x \leq -1 \\ 2 + [x] + [-x], & -1 < x < 1 \\ -x^2, & x \geq 1 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} -x^2, & x \leq -1 \\ 1, & -1 < x < 0 \\ 2, & x = 0 \\ 1, & 0 < x < 1 \\ -x^2, & x \geq 1 \end{cases}$$

Thus $f(x)$ is an even function.

$$\begin{aligned} \Rightarrow \int_{-2}^2 f(x) dx &= 2 \int_0^2 f(x) dx = 2 \left(\int_0^1 f(x) dx + \int_1^2 f(x) dx \right) \\ &= 2 \left(1 - \frac{x^3}{3} \Big|_0^1 \right) = 2 \left(1 - \frac{7}{3} \right) = -\frac{8}{3} \end{aligned}$$

19. (b) : The tangent drawn to $y = \sin x$ at $x = 0$ is the line $y = x$.

Clearly, the line $y = x$ lies above the graph of

$$y = \sin x \quad \forall x \in \left(0, \frac{\pi}{2}\right)$$

Thus required area

$$\Delta = \int_0^{\pi/2} (x - \sin x) dx = \left(\frac{x^2}{2} + \cos x \right) \Big|_0^{\pi/2} = \frac{\pi^2}{8} - 8 \text{ sq. units}$$

20. (d) : Let centre be $(h, 0)$ and radius be r .

It's equation is, $(x - h)^2 + y^2 = r^2$

Differentiating (i) w.r.t. x , we get

$$(x - h) + y \cdot \frac{dy}{dx} = 0$$

Differentiating both sides w.r.t. x again, we get

$$1 + y \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$$

21. (c) : Equation of any line drawn through the intersecting point of the given lines is,

$$3x + 4y - 12 + \lambda(x + 2y - 5) = 0$$

$$\Rightarrow A = \left(\frac{5\lambda + 12}{3 + \lambda}, 0 \right), B = \left(0, \frac{5\lambda + 12}{2\lambda + 4} \right)$$

If midpoint of AB is $C(h, k)$, then

$$2h = \frac{5\lambda + 12}{3 + \lambda}, 2k = \frac{5\lambda + 12}{2\lambda + 4}$$

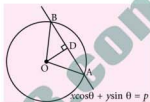
$$\Rightarrow \frac{h}{k} = \frac{2(\lambda + 2)}{3 + \lambda} \Rightarrow \lambda = \frac{4k - 3h}{(h - 2k)}$$

$$\text{Thus, } 2h = \frac{12 + \frac{5(4k - 3h)}{(h - 2k)}}{3 + \frac{4k - 3h}{h - 2k}}$$

Thus locus is $3x + 4y = 4xy$

22. (b) : If $\angle AOB = \frac{\pi}{2}$, then

$$OD = OA \cdot \cos \frac{\pi}{4} = \frac{a}{\sqrt{2}}$$



$$\Rightarrow \frac{a}{\sqrt{2}} = |p| \Rightarrow a^2 = 2p^2$$

23. (c) : Let the midpoint be $P(h, k)$.

Equation of this chord is $T = S_1$.

i.e., $yk - 2a(x + h) = k^2 - 4ah$

It must pass through $(a, 0)$

$$\Rightarrow -2a(a + h) = k^2 - 4ah$$

Thus required locus is

$$y^2 = 2ax - 2a^2$$

24. (c) : Tangent in terms of ' m ' for $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

$$\text{and for } \frac{x^2}{8} + \frac{y^2}{2} = 1 \text{ tangent is, } y = mx \pm \sqrt{8m^2 + 2}$$

$$\text{For common tangents we must have } \frac{1}{m} = \pm \sqrt{8m^2 + 2}$$

$$\Rightarrow m = \pm \frac{1}{2}$$

Thus, common tangents are $y = \frac{x}{2} + 2$ and $y = -\frac{x}{2} - 2$

i.e., $2y - x - 4 = 0$ and $2y + x + 4 = 0$

25. (b) : Slope of tangent should be ± 1 .

Thus, equation of tangent is $y = \pm x \pm \sqrt{a^2 + b^2}$

$$\text{It's distance from origin} = \frac{\sqrt{a^2 + b^2}}{\sqrt{2}} = \sqrt{\frac{a^2 + b^2}{2}}$$

26. (a) : Any point on the line is

$$P \equiv (6r_1 + 2, 3r_1 + 3, -4r_1 - 4)$$

Direction ratios of the line segment PQ, where

$Q \equiv (-1, 2, 6)$, are $6r_1 + 3, 3r_1 + 1, -4r_1 - 10$.

If 'P' be the foot of altitude drawn from Q to the given line, then

$$6(6r_1 + 3) + 3(3r_1 + 1) + 4(4r_1 + 10) = 0$$

$$\Rightarrow r_1 = -1$$

Thus, $P \equiv (-4, 0, 0)$

$\therefore$ Required distance $= \sqrt{9 + 4 + 36} = 7$ units

27. (d) : Given $\frac{\sin x}{\sin y} = \frac{1}{2}, \frac{\cos x}{\cos y} = \frac{3}{2} \Rightarrow \frac{\tan x}{\tan y} = \frac{1}{3}$

$$\text{Now, } \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = \frac{4 \tan x}{1 - 3 \tan^2 x} \quad \dots(i)$$

$$\text{Also, } \sin y = 2 \sin x, \cos y = \frac{2}{3} \cos x$$

$$\Rightarrow \sin^2 y + \cos^2 y = 4 \sin^2 x + \frac{4 \cos^2 x}{9} = 1$$

$$\Rightarrow 36 \tan^2 x + 4 = 9 \sec^2 x = 9(1 + \tan^2 x)$$

$$\Rightarrow 27 \tan^2 x = 5 \Rightarrow \tan x = \frac{\sqrt{5}}{3\sqrt{3}}$$

$$\therefore \text{ From (i), } \tan(x+y) = \frac{\frac{4\sqrt{5}}{3\sqrt{3}}}{1 - \frac{15}{27}} = \frac{4\sqrt{5} \cdot 27}{12 \cdot 3\sqrt{3}} = \sqrt{15}$$

28. (c) : $[\cot^{-1}x] + [\cos^{-1}x] = 0$

$$\Rightarrow [\cot^{-1}x] = [\cos^{-1}x] = 0$$

$$[\because \cos^{-1}x \geq 0 \text{ and } \cot^{-1}x > 0]$$

Now, $[\cot^{-1}x] = 0$

$$\Rightarrow x \in (\cot 1, \infty) \text{ and } [\cos^{-1}x] = 0$$

$$\Rightarrow x \in (\cos 1, 1) \Rightarrow x \in (\cot 1, 1)$$

29. (a) : Consider $1 - \tan \frac{B}{2} \cdot \tan \frac{C}{2}$

$$= 1 - \frac{\sqrt{\frac{(s-a)(s-b)}{s(s-b)}} \cdot \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}}{\frac{2a}{a+b+c}} = 1 - \frac{s-a}{s} = \frac{a}{s}$$

30. (a) : We must have

$$ax^3 + (a+b)x^2 + (b+c)x + c > 0$$

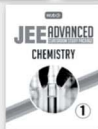
$$\Rightarrow ax^2(x+1) + bx(x+1) + c(x+1) > 0$$

$$\Rightarrow (x+1)(ax^2 + bx + c) > 0$$

$$\Rightarrow a(x+1)(ax^2 + bx + c) > 0$$

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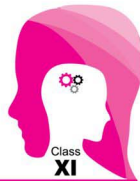
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Complex Numbers



Class
XI

This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

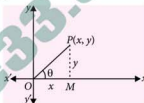
* ALOK KUMAR, B.Tech, IIT Kanpur

- Any number which can be expressed in the form $x + iy$, where x, y are real numbers and $i = \sqrt{-1}$ is called a complex number.
 $i^2 = -1, i^3 = i^2 i = -i, i^4 = 1.$
- In general, $i^{4n} = 1, i^{4n+1} = i, i^{4n+2} = -1, i^{4n+3} = -i$ for an integer n .
- If $z = x + iy$, then the real part of z is denoted by $\text{Re}(z)$ and the imaginary part by $\text{Im}(z)$.
- A complex number is said to be purely real if $\text{Im}(z) = 0$, and is said to be purely imaginary if $\text{Re}(z) = 0$. The complex number $0 = 0 + i0$ is both purely real and purely imaginary.
- Two complex numbers are said to be equal if and only if their real parts and imaginary parts are separately equal i.e., $a + ib = c + id \Rightarrow a = c$ and $b = d$. However, there is no order relation between complex numbers and the expressions of the form $a + ib < (\text{or}) > c + id$ are meaningless.
- Since a real number a can be written as $a + i0$, therefore every real number can be considered as a complex number whose imaginary part is zero. Thus the set R of real numbers is a proper subset of the complex numbers C .

REPRESENTATION OF COMPLEX NUMBERS IN ARGAND PLANE

A complex number $z = x + iy$ written as ordered pair (x, y) can be represented by a point P whose Cartesian coordinates are (x, y) referred to axes OX and OY , usually called the real and the imaginary axes. The

plane of OX and OY is called the Argand diagram or the complex plane.



MODULUS OF A COMPLEX NUMBER

Let $z = x + iy$ be a complex number then its magnitude is defined by the real number $\sqrt{x^2 + y^2}$ and is denoted by $|z|$.

ARGUMENT OF A COMPLEX NUMBER

- If $z = x + iy$, then $\tan \theta = \frac{y}{x}$ is said to be the argument or amplitude of the complex number z and is denoted by $\arg(z)$ or $\text{amp}(z)$. In case of $x = 0$ (where $y \neq 0$), $\arg(z) = +\pi/2$ or $-\pi/2$ depending upon $y > 0$ or $y < 0$ and the complex number is called purely imaginary. If $y = 0$ (where $x \neq 0$), then $\arg(z) = 0$ or π depending upon $x > 0$ or $x < 0$ and the complex number is called purely real. The argument of the complex number 0 is not defined.

We can define the argument of a complex number also as any value of the θ which satisfies the system

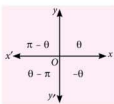
$$\text{of equations } \cos \theta = \frac{x}{\sqrt{x^2 + y^2}}, \sin \theta = \frac{y}{\sqrt{x^2 + y^2}}.$$

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91).
He trains IIT and Olympiad aspirants.

- **Principal Argument of a Complex Number:** The value of θ satisfying the inequality $-\pi < \theta \leq \pi$ is called the principal value of the argument.
- Method of finding the principal argument of a complex number $z = x + iy$.

Find $\tan \theta = \frac{y}{x}$ and this gives the value of θ .

Find the quadrant in which z lies, with the help of sign of x and y coordinates. Then argument of z will be $\theta, \pi - \theta, \theta - \pi$ and $-\theta$ according as z lies in the first, second, third or fourth quadrant.



POLAR FORM OF A COMPLEX NUMBER

Let $OP = r$, then $x = r \cos \theta$ and $y = r \sin \theta$

$\therefore z = x + iy = r \cos \theta + ir \sin \theta = r(\cos \theta + i \sin \theta)$.

This is known as Polar form (or Trigonometric form) of a Complex Number. Here we should take the principal value of θ . For general values of the argument $z = r [\cos(2n\pi + \theta) + i \sin(2n\pi + \theta)]$, $\forall n \in \mathbb{I}$.

Sometimes $\cos \theta + i \sin \theta$ is, in short, written as $\text{cis}(\theta)$.

- **Euler's Formula:** $\cos \theta + i \sin \theta = e^{i\theta}$.
- When complex numbers are multiplied, their arguments get added.
 $(\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2) = \cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)$

Generalizing, $(\cos \theta_1 + i \sin \theta_1)(\cos \theta_2 + i \sin \theta_2) \dots$

$(\cos \theta_n + i \sin \theta_n) = \cos(\theta_1 + \theta_2 + \dots + \theta_n) + i \sin(\theta_1 + \theta_2 + \dots + \theta_n)$

- When complex numbers are divided, their arguments get subtracted.

$\frac{(\cos \theta_1 + i \sin \theta_1)}{(\cos \theta_2 + i \sin \theta_2)} = \cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)$

- **Unimodular Complex Number:**

A complex number z such that $|z| = 1$ is said to be unimodular complex number.

Clearly z lies on a circle of radius 1 unit and centre $(0, 0)$.

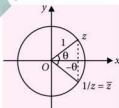
If $|z| = 1 \Rightarrow z = \cos \theta + i \sin \theta$,

$\Rightarrow 1/z = (\cos \theta + i \sin \theta)^{-1} = \cos \theta - i \sin \theta$

- **Algebraic Operations with Complex Numbers:**

➤ **Addition:** $(a + ib) + (c + id) = (a + c) + i(b + d)$

➤ **Subtraction:** $(a + ib) - (c + id) = (a - c) + i(b - d)$



➤ **Multiplication:** $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

➤ **Division:** $\frac{a + ib}{c + id}$ (when at least one of c and d

is non-zero) $= \frac{(ac + bd)}{c^2 + d^2} + \frac{i(bc - ad)}{c^2 + d^2}$

SQUARE ROOT OF A COMPLEX NUMBER

Let $z_1 = x_1 + iy_1$ be a complex number such that

$x + iy = (x_1 + iy_1)^{1/2}$

$\Rightarrow x^2 - y^2 + 2ixy = x_1 + iy_1$

$\Rightarrow x_1 = x^2 - y^2$ and $y_1 = 2xy$

$\Rightarrow x_1 = x^2 - \frac{y_1^2}{4x^2}$

$\Rightarrow x^2 = \frac{|z_1| + x_1}{2}, y = \frac{\pm y_1}{|y_1|} \sqrt{\frac{|z_1| - x_1}{2}}$

If $y_1 > 0$, then $x = \pm \sqrt{\frac{|z_1| + x_1}{2}}, y = \pm \sqrt{\frac{|z_1| - x_1}{2}}$ and

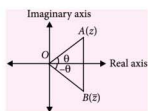
$\sqrt{x_1 + iy_1} = \pm \left(\sqrt{\frac{(\text{Re}(z_1) + |z_1|)}{2}} + i \sqrt{\frac{|z_1| - \text{Re}(z_1)}{2}} \right)$.

If $y_1 < 0$, then $x = \pm \sqrt{\frac{|z_1| + x_1}{2}}, y = \mp \sqrt{\frac{|z_1| - x_1}{2}}$ and

$\sqrt{x_1 + iy_1} = \pm \left(\sqrt{\frac{(\text{Re}(z_1) + |z_1|)}{2}} - i \sqrt{\frac{|z_1| - \text{Re}(z_1)}{2}} \right)$.

CONJUGATE OF A COMPLEX NUMBER

- The conjugate of the complex number $z = a + ib$ is defined to be $a - ib$ and is denoted by $\bar{z}$. In other words $\bar{z}$ is the mirror image of z in the real axis.



- **Properties of Conjugate of a Complex Number:**

Let $z = x + iy$, then

➤ $\overline{\bar{z}} = z$

➤ $|z| = |\bar{z}|$

➤ $z + \bar{z} = 2\text{Re}(z), z - \bar{z} = 2i\text{Im}(z)$

➤ If z is purely real, then $z = \bar{z}$.

➤ If z is purely imaginary, then $z + \bar{z} = 0$.

➤ $z\bar{z} = |z|^2 = |\bar{z}|^2$

➤ $z_1 + z_2 = \bar{z}_1 + \bar{z}_2$

In general, $\overline{z_1 + z_2 + \dots + z_n} = \overline{z_1} + \overline{z_2} + \dots + \overline{z_n}$

$$\triangleright \overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$$

$$\triangleright \overline{z_1 z_2} = \overline{z_1} \cdot \overline{z_2}$$

In general, $\overline{z_1 z_2 z_3 \dots z_n} = \overline{z_1} \cdot \overline{z_2} \cdot \overline{z_3} \dots \overline{z_n}$

$$\triangleright \overline{(z^n)} = (\overline{z})^n$$

$$\triangleright \overline{\left(\frac{z_1}{z_2}\right)} = \left(\frac{\overline{z_1}}{\overline{z_2}}\right), (z_2 \neq 0)$$

• Properties of Modulus :

Let $z = x + iy$, then

$$\triangleright |z| = 0 \Rightarrow z = 0 + i0$$

$$\triangleright |z_1 - z_2| \text{ denotes the distance between } z_1 \text{ and } z_2.$$

$$\triangleright -|z| \leq \operatorname{Re}(z) \leq |z|; \text{ equality holds on right or on left side depending upon } z \text{ being positive real or negative real.}$$

$$\triangleright -|z| \leq \operatorname{Im}(z) \leq |z|; \text{ equality holds on right side or on left side depending upon } z \text{ being purely imaginary and above the real axis or below the real axis.}$$

$$\triangleright |z| \leq |\operatorname{Re}(z)| + |\operatorname{Im}(z)| \leq \sqrt{2} |z|; \text{ equality holds on left side when } z \text{ is purely imaginary or purely real and equality holds on right side when } |\operatorname{Re}(z)| = |\operatorname{Im}(z)|.$$

$$\triangleright |z|^2 = z\overline{z} \Rightarrow \text{Purely real}$$

$$\triangleright |z_1 z_2| = |z_1| |z_2|$$

In general, $|z_1 z_2 \dots z_n| = |z_1| |z_2| \dots |z_n|$

$$\triangleright |z^n| = |z|^n, n \in I$$

$$\triangleright \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, (z_2 \neq 0)$$

$$\triangleright |z_1 + z_2| \leq |z_1| + |z_2| \Rightarrow |z_1 + z_2 + \dots + z_n| \leq |z_1| + |z_2| + \dots + |z_n|; \text{ equality holds if origin, } z_1, z_2, z_3, \dots, z_n \text{ are collinear and } z_1, z_2, z_3, \dots, z_n \text{ are on the same side of the origin.}$$

$$\triangleright |z_1 - z_2| \geq ||z_1| - |z_2||; \text{ equality holds when } \arg(z_1/z_2) = \pi \text{ i.e., origin, } z_1, z_2 \text{ are collinear and } z_1 \text{ and } z_2 \text{ are on the opposite side of the origin.}$$

$$\begin{aligned} \triangleright |z_1 + z_2|^2 &= (z_1 + z_2)(\overline{z_1} + \overline{z_2}) \\ &= |z_1|^2 + |z_2|^2 + z_1 \overline{z_2} + z_2 \overline{z_1} \\ &= |z_1|^2 + |z_2|^2 + 2 \operatorname{Re}(z_1 \overline{z_2}) \end{aligned}$$

$$\begin{aligned} \triangleright |z_1 - z_2|^2 &= (z_1 - z_2)(\overline{z_1} - \overline{z_2}) \\ &= |z_1|^2 + |z_2|^2 - z_1 \overline{z_2} - z_2 \overline{z_1} \\ &= |z_1|^2 + |z_2|^2 - 2 \operatorname{Re}(z_1 \overline{z_2}) \end{aligned}$$

Properties of Argument :

$$\bullet \arg(z_1 z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$$

$$\bullet \arg(z_1/z_2) = \theta_1 - \theta_2 = \arg(z_1) - \arg(z_2)$$

$$\bullet \arg(z^n) = n \arg(z), n \in I$$

$$\bullet \arg(z) = 0, \pi \Rightarrow z \text{ is a purely real number } \Rightarrow z = \overline{z}.$$

$$\bullet \arg(z) = \pi/2, -\pi/2 \Rightarrow z \text{ is a purely imaginary number } \Rightarrow z = -\overline{z}.$$

DE-MOIVRE'S THEOREM

If n is any integer, then $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$. This is known as De-Moivre's Theorem.

• Writing the binomial expansion of $(\cos\theta + i \sin\theta)^n$ and equating the real part to $\cos n\theta$ and the imaginary part to $\sin n\theta$, we get

$$\begin{aligned} \cos n\theta &= \cos^n \theta - {}^nC_2 \cos^{n-2} \theta \sin^2 \theta \\ &\quad + {}^nC_4 \cos^{n-4} \theta \sin^4 \theta - \dots \\ \sin n\theta &= {}^nC_1 \cos^{n-1} \theta \sin \theta - {}^nC_3 \cos^{n-3} \theta \sin^3 \theta \\ &\quad + {}^nC_5 \cos^{n-5} \theta \sin^5 \theta - \dots \end{aligned}$$

$$\therefore \tan n\theta = \frac{{}^nC_1 \tan \theta - {}^nC_3 \tan^3 \theta + {}^nC_5 \tan^5 \theta - \dots}{1 - {}^nC_2 \tan^2 \theta + {}^nC_4 \tan^4 \theta - \dots}$$

• If n is rational number, then one of the values of $(\cos\theta + i \sin\theta)^n$ is $\cos n\theta + i \sin n\theta$. Let $n = p/q$, where p and q are integers ($q > 0$) and p, q have no common factors, then $(\cos\theta + i \sin\theta)^q$ has q distinct roots, one of which is $\cos n\theta + i \sin n\theta$.

• If $z = r(\cos\theta + i \sin\theta)$ and n is a positive integer, then $z^{1/n} = r^{1/n} \left[\cos\left(\frac{2k\pi + \theta}{n}\right) + i \sin\left(\frac{2k\pi + \theta}{n}\right) \right]$, where $k = 0, 1, 2, \dots, n-1$.

n^{th} ROOTS OF UNITY

• Let x be the n^{th} root of unity, then

$$x^n = 1 = \cos 2k\pi + i \sin 2k\pi \quad \forall k \in I.$$

$$\Rightarrow x = \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}; k = 0, 1, 2, \dots, n-1$$

$$\text{Let } \alpha = \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$$

When $k = 2$:

$$\begin{aligned} \cos \frac{4\pi}{n} + i \sin \frac{4\pi}{n} &= \cos\left(\frac{2 \times 2\pi}{n}\right) + i \sin\left(\frac{2 \times 2\pi}{n}\right) \\ &= \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}\right)^2 = \alpha^2 \end{aligned}$$

When $k = 3$:

$$\cos \frac{6\pi}{n} + i \sin \frac{6\pi}{n} = \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}\right)^3 = \alpha^3$$

Similarly, when $k = t$:

$$\text{Then, } \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} = \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n} \right)^k = \alpha^k$$

$$\therefore n^{\text{th}} \text{ roots of unity are } 1, \alpha, \alpha^2, \dots, \alpha^{n-1}.$$

• **Sum of Roots :**

$$1 + \alpha + \alpha^2 + \dots + \alpha^{n-1} = \frac{1 - \alpha^n}{1 - \alpha} = 0 \quad (\because \alpha^n = 1)$$

$$\Rightarrow \sum_{k=0}^{n-1} \cos \frac{2k\pi}{n} = 0 \text{ and } \sum_{k=0}^{n-1} \sin \frac{2k\pi}{n} = 0$$

Thus the sum of n^{th} roots of unity is always zero.

• **Product of Roots :**

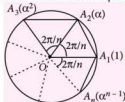
$$1 \cdot \alpha \cdot \alpha^2 \dots \alpha^{n-1} = \alpha^{\frac{n(n-1)}{2}} = \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n} \right)^{\frac{n(n-1)}{2}}$$

$$= \cos \left\{ \pi(n-1) \right\} + i \sin \left\{ \pi(n-1) \right\}$$

$$\text{If } n \text{ is even, then } \alpha^{\frac{n(n-1)}{2}} = -1$$

$$\text{If } n \text{ is odd, then } \alpha^{\frac{n(n-1)}{2}} = 1$$

- n^{th} roots of unity lie on the circle of unit radii and divide the circumference into n equal parts.



CUBE ROOTS OF UNITY

Cube roots of unity are solution set of the equation $x^3 - 1 = 0$.

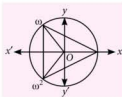
$$\Rightarrow x = 1^{1/3} \Rightarrow x = (\cos 0 + i \sin 0)^{1/3}$$

$$\Rightarrow x = \cos \left(\frac{2k\pi}{3} \right) + i \sin \left(\frac{2k\pi}{3} \right), \text{ where } k = 0, 1, 2$$

$$\therefore \text{Roots are } 1, \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \text{ and } \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$$

$$\text{i.e. } 1, \frac{-1+i\sqrt{3}}{2} \text{ and } \frac{-1-i\sqrt{3}}{2}.$$

They are generally denoted by $1, \omega$ and ω^2 and geometrically represented by the vertices of an equilateral triangle whose circumcentre is origin and circumradius is unity.



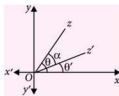
$$\triangleright \omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\triangleright 1 + \omega^n + \omega^{2n} = 3 \text{ (if } n \text{ is a multiple of 3)}$$

$$1 + \omega^n + \omega^{2n} = 0 \text{ (if } n \text{ is an integer, not a multiple of 3)}$$

ROTATION IN COMPLEX NUMBERS

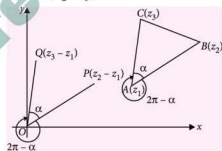
- If z and z' are two complex numbers, then argument of $\frac{z}{z'}$ is the angle through which Oz' must be turned in order that it may lie along Oz .



$$\frac{z}{z'} = \frac{|z|e^{i\theta}}{|z'|e^{i\theta'}} = \frac{|z|}{|z'|}e^{i(\theta-\theta')} = \frac{|z|}{|z'|}e^{i\alpha}$$

In general, let z_1, z_2, z_3 be the three vertices of a triangle ABC described in the counter-clockwise sense. Draw OP and OQ parallel and equal to AB and AC respectively. Then the point P is $z_2 - z_1$ and Q is $z_3 - z_1$ and $\frac{z_3 - z_1}{z_2 - z_1} = \frac{OQ}{OP}(\cos \alpha + i \sin \alpha)$

$$= \frac{CA}{BA}e^{i\alpha} = \frac{|z_3 - z_1|}{|z_2 - z_1|}e^{i\alpha}$$



Note that $\arg(z_3 - z_1) - \arg(z_2 - z_1) = \alpha$ is the angle through which OP must be rotated in the anti-clockwise direction so that it becomes parallel to OQ .

$$\text{Here, we can write } \frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|}e^{-i(2\pi - \alpha)}$$

also. In this case we are rotating OP in clockwise direction by an angle $(2\pi - \alpha)$. Since the rotation is in clockwise direction, we are taking negative sign with angle $(2\pi - \alpha)$.

GEOMETRY OF COMPLEX NUMBERS

- **Section Formula:** Let z_1 and z_2 be any two complex numbers representing the points A and B respectively in the argand plane. Let C be the point dividing the line segment AB internally in the ratio

$m : n$ i.e., $\frac{AC}{BC} = \frac{m}{n}$ and let the complex number

associated with point C be z .

Let us rotate the line BC about the point C so that it becomes parallel to CA. The corresponding equation of rotation will be

$$\frac{z - z_1}{z - z_2} = \frac{m}{n}(-1)$$

$$\Rightarrow nz_1 - nz = -mz_2 + mz \Rightarrow z = \frac{mz_2 + nz_1}{m+n}$$

Similarly, if $C(z)$ divides the line segment AB

externally in the ratio $m : n$, then $z = \frac{mz_2 - nz_1}{m - n}$.

- If $C(z)$ is the mid point of AB, then $z = \frac{z_1 + z_2}{2}$.
- **Condition for Collinearity:** If there are three real numbers (other than 0) l, m and n such that $lz_1 + mz_2 + nz_3 = 0$ and $l + m + n = 0$, then complex numbers z_1, z_2 and z_3 will be collinear.

Equation of a Straight Line :

➤ Equation of the line through z_1 and z_2 is given by

$$\frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1} \quad \text{or} \quad \begin{vmatrix} z & \bar{z} \\ z_1 & \bar{z}_1 \\ z_2 & \bar{z}_2 \end{vmatrix} = 0.$$

➤ General equation of a straight line is $a\bar{z} + \bar{a}z + b = 0$, where a is a non zero complex number and b is any real number.

➤ The length of perpendicular from a point z_1 to the line $a\bar{z} + \bar{a}z + b = 0$ is given by $\frac{|a\bar{z}_1 + \bar{a}z_1 + b|}{2|a|}$

Equation of a Circle :

Consider a fixed complex number z_0 and let z be any complex number which moves in such a way that its distance from z_0 is always equals to ' r '. This implies z would lie on a circle whose centre is z_0 and radius is r . So its equation would be $|z - z_0| = r$.

$$\Rightarrow |z - z_0|^2 = r^2$$

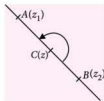
$$\Rightarrow (z - z_0)(\bar{z} - \bar{z}_0) = r^2$$

$$\Rightarrow z\bar{z} - z\bar{z}_0 - \bar{z}z_0 + z_0\bar{z}_0 - r^2 = 0$$

➤ Let $z_0 = -a$ and $z_0\bar{z}_0 - r^2 = b$

$$\Rightarrow z\bar{z} + a\bar{z} + \bar{a}z + b = 0,$$

where centre $= -a$ and radius $= \sqrt{|a|^2 - b}$.



PROBLEMS

Single Correct Answer Type

- $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary, if $\theta =$
 - $2n\pi \pm \frac{\pi}{3}$
 - $n\pi \pm \frac{\pi}{3}$
 - $n\pi \pm \frac{\pi}{3}$
 - None of these
- If $z = 3 - 4i$, then $z^4 - 3z^3 + 3z^2 + 99z - 95$ is equal to
 - 5
 - 6
 - 5
 - 4
- $A + iB$ form of $\frac{(\cos x + i\sin x)(\cos y + i\sin y)}{(\cot u + i)(1 + i\tan v)}$ is
 - $\sin u \cos v [\cos(x + y - u - v) + i\sin(x + y - u - v)]$
 - $\sin u \cos v [\cos(x + y + u + v) + i\sin(x + y + u + v)]$
 - $\sin u \cos v [\cos(x + y + u + v) - i\sin(x + y + u + v)]$
 - None of the above
- If $z = x + iy$, $z^{1/3} = a - ib$ and $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$, then value of k is equals to
 - 2
 - 4
 - 6
 - 1
- If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) =$
 - $A^2 + B^2$
 - $A^2 - B^2$
 - A^2
 - B^2
- If $\frac{2z_1}{3z_2}$ is a purely imaginary number, then $\left| \frac{z_1 - z_2}{z_1 + z_2} \right| =$
 - 3/2
 - 1
 - 2/3
 - 4/9
- If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then
 - $|z| = 0$
 - $|z| = 1$
 - $|z| > 1$
 - $|z| < 1$
- The solution of the equation $|z| - z = 1 + 2i$ is
 - $2 - \frac{3}{2}i$
 - $\frac{3}{2} + 2i$
 - $\frac{3}{2} - 2i$
 - $-2 + \frac{3}{2}i$
- If z_1, z_2 are two complex numbers such that $\left| \frac{z_1 - z_2}{z_1 + z_2} \right| = 1$ and $iz_1 = kz_2$, where $k \in \mathbb{R}$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is
 - $\tan^{-1} \left(\frac{2k}{k^2 + 1} \right)$
 - $\tan^{-1} \left(\frac{2k}{1 - k^2} \right)$
 - $-2\tan^{-1} k/2$
 - $2\tan^{-1} k$
- $\left| \frac{1}{2}(z_1 + z_2) + \sqrt{z_1 z_2} \right| + \left| \frac{1}{2}(z_1 + z_2) - \sqrt{z_1 z_2} \right| =$

CONCEPT
MAP

STATISTICS

Class XI Class XII

MATRICES

CONCEPT
MAP

Measures of Central Tendency

Mean, Median, Mode

Analysis of Frequency Distributions

Coefficient of variation (C.V.) = $\frac{S.D.}{\text{Mean}} \times 100 = \frac{\sigma}{\bar{x}} \times 100$, $\bar{x} \neq 0$

- The data having higher C.V. will be more variable and the data having lower C.V. will be more consistent.

For Ungrouped Data

Let $x_1, x_2, \dots, x_n$ be n observations, then mean deviation

- About mean = $\frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|$, where $\bar{x}$ = Mean

- About median = $\frac{1}{n} \sum_{i=1}^n |x_i - M|$, where M = Median

For Grouped Data

Let $x_1, x_2, \dots, x_n$ be a set of n observations occurring with frequencies $f_1, f_2, \dots, f_n$ respectively, then mean deviation

- About mean, $\bar{x} = \frac{1}{N} \sum_{i=1}^n f_i |x_i - \bar{x}|$

- About median, $M = \frac{1}{N} \sum_{i=1}^n f_i |x_i - M|$

Here, x_i are the mid-points of classes and $N = \sum_{i=1}^n f_i$ = Sum of frequencies.

Short Cut Method

- Mean deviation about mean, $\bar{x} = \frac{1}{N} \sum_{i=1}^n f_i |x_i - \bar{x}|$

where mean, $(\bar{x}) = a + \frac{\sum f_i u_i}{N} \times h$, where $u_i = \frac{x_i - a}{h}$, a is assumed mean and h is width of class-intervals.

- Mean deviation about median, $M = \frac{1}{N} \sum_{i=1}^n f_i |x_i - M|$

where median, $(M) = J + \frac{\frac{N}{2} - cf}{f} \times h$, where h , f and cf are lower limit, frequency and cumulative frequency.

Measures of Dispersion

Range

Difference between two extreme observations of the given distribution.
Range = Maximum Value - Minimum Value

Quartiles

Quartiles are those values of the variate which divide the distribution into four equal parts.

Mean Deviation

It is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode).

Mean Deviation = $\frac{\text{Sum of Deviations}}{\text{Number of Observations}}$

Standard Deviation

Positive square root of the variance is known as standard deviation and it is denoted by σ .

For ungrouped data

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

For grouped data

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2}$$

Shortcut method

$$\sigma = \sqrt{\frac{h}{N} \left[\sum_{i=1}^n f_i u_i^2 - \left(\sum_{i=1}^n f_i u_i \right)^2 \right]}$$

where $u_i = \frac{x_i - a}{h}$, a = assumed mean, h = width of class-intervals

Variance of Combined Series

Let $\bar{x}_1$ and $\bar{x}_2$ be means of two groups with standard deviations σ_1 and σ_2 and number of elements n_1 and n_2 then variance is given by

$$(\sigma^2) = \frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2}$$

where $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$ andhttps://t.me/Estore33_com

Matrix

Any rectangular arrangement of numbers is called a matrix.

Order of a Matrix

- A matrix having m rows and n columns is called a matrix of order $m \times n$.

Transpose of a Matrix

Transpose of a matrix is obtained by interchanging rows and columns.

- If $A = [a_{ij}]_{m \times n}$, then A' or $A^T = [a_{ji}]_{n \times m}$
- $(A')' = A$
- $(AB)' = B' A'$
- $(kA)' = kA'$

Types of Matrices

- Column Matrix: $A = [a_{ij}]_{m \times 1}$
- Row Matrix: $A = [a_{ij}]_{1 \times n}$
- Square Matrix: $A = [a_{ij}]_{n \times n}$
- Diagonal Matrix: $A = [a_{ij}]_{n \times n}$, where $a_{ij} = 0 \forall i \neq j$
- Zero Matrix: $A = [a_{ij}]_{m \times n}$, where $a_{ij} = 0 \forall i$ and j
- Identity Matrix: $A = [a_{ij}]_{n \times n}$, where $a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$
- Involuntary Matrix: $A^2 = I$
- Orthogonal Matrix: $AA^T = A^T A = I$
- Idempotent Matrix: $A^2 = A$
- Unitary Matrix: $AA^T = A^T A = I$
- Symmetric Matrix: $A^T = A$
- Skew-Symmetric Matrix: $A^T = -A$

Elementary Operations

- $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
- $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$
- $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$

Inverse of a Matrix

- A unique matrix B for A such that $AB = BA = I$.
- Inverse of a square matrix, if it exists, is unique.

$$(AB)^{-1} = B^{-1} A^{-1}$$

$$(A^{-1})^{-1} = (A^{-1})^T$$

Operations on Matrices

Addition and Subtraction: $A \pm B = C$ i.e., $[a_{ij}]_{m \times n} \pm [b_{ij}]_{m \times n} = [a_{ij} \pm b_{ij}]_{m \times n} = [c_{ij}]_{m \times n}$

$$A + B = B + A$$

$$A + (B + C) = (A + B) + C$$

$$\text{Additive inverse} = -A$$

$$\text{Additive identity} = O$$

Multiplication: $A_{m \times n} \times B_{n \times p} = C_{m \times p}$

$$\text{i.e., } \left[\sum_{k=1}^n a_{ik} b_{kj} \right] = [c_{ij}]$$

- AB exist $\Rightarrow BA$ exists
- AB may or may not be equal to BA
- $(AB)C = A(BC)$
- $I_m \times A_{m \times n} = A_{m \times n} = A_{m \times n} \times I_n$
- $A(B + C) = AB + AC$; $(B + C)A = BA + CA$
- $A(B \cdot C) = (AB)C$; $(B \cdot C)A = B(A \cdot C)$
- $k(A + B) = kA + kB$
- $(k + m)A = kA + mA$

Trace of a Matrix

Sum of principal diagonal elements of a square matrix A is called trace of a matrix.

- $\text{tr}(A) + \text{tr}(B) = \text{tr}(A + B)$
- $\text{tr}(kA) = k \text{tr}(A)$

Comparable Matrices

Two matrices are said to be comparable if they have same orders.

Equal Matrices

Two comparable matrices A and B are said to be equal if all of their corresponding elements are equal.

- (a) $|z_1 + z_2|$ (b) $|z_1 - z_2|$
 (c) $|z_1| + |z_2|$ (d) $|z_1| - |z_2|$

11. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$, where $z \neq -1$, then $\text{Re}(\omega)$ is equal to

- (a) 0 (b) $-\frac{1}{|z+1|^2}$
 (c) $\frac{|z|}{|z+1|} \cdot \frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

12. A real value of x will satisfy the equation

$$\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta (\alpha, \beta \text{ real}), \text{ if}$$

- (a) $\alpha^2 - \beta^2 = -1$ (b) $\alpha^2 - \beta^2 = 1$
 (c) $\alpha^2 + \beta^2 = 1$ (d) $\alpha^2 - \beta^2 = 2$

13. If $(\sqrt{8} + i)^{50} = 3^{49}(\alpha + i\beta)$, then $\alpha^2 + \beta^2 =$

- (a) 3 (b) 8 (c) 9 (d) $\sqrt{8}$

14. The number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$ is

- (a) infinite (b) 1
 (c) 2 (d) None of these

15. If $(1 + i\sqrt{3})^9 = a + ib$, then b is equal to

- (a) 1 (b) 256 (c) 0 (d) 9^3

16. If $x + \frac{1}{x} = \sqrt{3}$, then $x =$

- (a) $\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$ (b) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$
 (c) $\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}$ (d) $\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$

17. POQ is a straight line through the origin O. P and Q represent the complex numbers $a + ib$ and $c + id$ respectively and $OP = OQ$, then

- (a) $|a + ib| = |c + id|$ (b) $a + c + b + d = 0$
 (c) $\arg(a + ib) = \arg(c + id)$ (d) None of these

18. Let a be a complex number such that $|a| < 1$ and $z_1, z_2, \dots$ be vertices of a polygon such that $z_k = 1 + a + a^2 + \dots + a^{k-1}$. Then the vertices of the polygon lie within a circle

- (a) $|z - a| = a$ (b) $\left|z - \frac{1}{1-a}\right| = |1-a|$
 (c) $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$ (d) $|z - (1-a)| = |1-a|$

19. If $\frac{|z-2|}{|z-3|} = 2$ represents a circle, then its radius is equal to

- (a) 1 (b) $1/3$ (c) $3/4$ (d) $2/3$

20. If z_1, z_2, z_3, z_4 are the affixes of four points in the argand plane and z is the affix of a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then z_1, z_2, z_3, z_4 are

- (a) concyclic
 (b) vertices of a parallelogram
 (c) vertices of a rhombus
 (d) in a straight line

21. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represents the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number

- (a) $3 - \frac{i}{2}$ or $1 - \frac{3}{2}i$ (b) $\frac{3}{2} - i$ or $\frac{1}{2} - 3i$
 (c) $\frac{1}{2} - i$ or $1 - \frac{1}{2}i$ (d) None of these

22. Let z_1, z_2, z_3 be three vertices of an equilateral triangle circumscribing the circle $|z| = \frac{1}{2}$. If

$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ and z_1, z_2, z_3 are in anticlockwise sense, then z_2 is

- (a) $1 + \sqrt{3}i$ (b) $1 - \sqrt{3}i$
 (c) i (d) -1

23. If z is a complex number in the argand plane, then the equation $|z - 2| + |z + 2| = 8$ represents

- (a) parabola (b) ellipse
 (c) hyperbola (d) circle

24. The points $1 + 3i, 5 + i$ and $3 + 2i$ in the complex plane are

- (a) vertices of a right angled triangle
 (b) collinear
 (c) vertices of an obtuse angled triangle
 (d) vertices of an equilateral triangle

25. If $z = x + iy$, then area of the triangle whose vertices are points z, iz and $z + iz$ is

- (a) $2|z|^2$ (b) $\frac{1}{2}|z|^2$ (c) $|z|^2$ (d) $\frac{3}{2}|z|^2$

26. If A, B, C are represented by $3 + 4i, 5 - 2i, -1 + 16i$, then A, B, C are

- (a) collinear
 (b) vertices of an equilateral triangle
 (c) vertices of isosceles triangle
 (d) vertices of right angled triangle

27. If z_1, z_2, z_3 are affixes of the vertices A, B and C respectively of a triangle ABC having centroid at G, such that $z = 0$ is the mid point of AG, then

- (a) $z_1 + z_2 + z_3 = 0$ (b) $z_1 + 4z_2 + z_3 = 0$
 (c) $z_1 + z_2 + 4z_3 = 0$ (d) $4z_1 + z_2 + z_3 = 0$

28. If $z_1 = 1 + i$, $z_2 = -2 + 3i$ and $z_3 = ai/3$, where $i^2 = -1$ are collinear, then the value of a is
 (a) -1 (b) 3 (c) 4 (d) 5

29. The area of the triangle whose vertices are represented by the complex numbers $0, z, ze^{i\alpha}$, ($0 < \alpha < \pi$) is equals to

- (a) $\frac{1}{2}|z|^2 \cos \alpha$ (b) $\frac{1}{2}|z|^2 \sin \alpha$
 (c) $\frac{1}{2}|z|^2 \sin \alpha \cos \alpha$ (d) $\frac{1}{2}|z|^2$

30. When $\frac{z+i}{z+2}$ is purely imaginary, the locus described by the point z in the argand diagram is a

- (a) circle of radius $\frac{\sqrt{5}}{2}$ (b) circle of radius $\frac{5}{4}$
 (c) straight line (d) parabola

31. If $|z+1| = \sqrt{2}|z-1|$, then the locus described by the point z in the argand diagram is a

- (a) straight line (b) circle
 (c) parabola (d) None of these

32. The region of the complex plane for which

$$\left| \frac{z-a}{z+\bar{a}} \right| = 1 \quad [\operatorname{Re}(a) \neq 0] \text{ is}$$

- (a) x-axis (b) y-axis
 (c) the straight line $x = a$ (d) None of these

33. The region of argand plane defined by $|z-1| + |z+1| \leq 4$ is

- (a) interior of an ellipse
 (b) exterior of a circle
 (c) interior and boundary of an ellipse
 (d) None of these

34. The locus of the point z which satisfy the condition

$$\arg \left(\frac{z-1}{z+1} \right) = \frac{\pi}{3} \text{ is}$$

- (a) a straight line (b) a circle
 (c) a parabola (d) None of these

35. If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then the locus of the point representing z in the complex plane is

- (a) a circle (b) a straight line
 (c) a parabola (d) None of these

36. $\operatorname{Re}(z^2) = 1$ is represented by

- (a) the circle $x^2 + y^2 = 1$ (b) the hyperbola $x^2 - y^2 = 1$
 (c) parabola or a circle (d) all the above

37. The locus represented by $|z-1| = |z+i|$ is

- (a) a circle of radius 1
 (b) an ellipse with foci at (1, 0) and (0, -1)
 (c) a straight line through the origin
 (d) a circle on the line joining (1, 0), (0, 1) as diameter

38. If $z = x + iy$ and $|z-2+i| = |z-3-i|$, then locus of z is

- (a) $2x + 4y - 5 = 0$ (b) $2x - 4y - 5 = 0$
 (c) $x + 2y = 0$ (d) $x - 2y + 5 = 0$

39. Locus of the point z satisfying the equation $|iz-1| + |z-i| = 2$ is

- (a) a straight line (b) a circle
 (c) an ellipse (d) a pair of straight lines

40. If $z = x + iy$ is a complex number satisfying

$$\left| z + \frac{i}{2} \right| = \left| z - \frac{i}{2} \right|, \text{ then the locus of } z \text{ is}$$

- (a) $2y = x$ (b) $y = x$ (c) y-axis (d) x-axis

41. If the amplitude of $z - 2 - 3i$ is $\pi/4$, then the locus of $z = x + iy$ is

- (a) $x + y - 1 = 0$ (b) $x - y - 1 = 0$
 (c) $x + y + 1 = 0$ (d) $x - y + 1 = 0$

42. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

- (a) an ellipse (b) the imaginary axis
 (c) a circle (d) the real axis

43. If $z = x + iy$ and $\omega = \frac{1-iz}{z-i}$, then $|\omega| = 1$ shows that in complex plane

- (a) z will be at imaginary axis
 (b) z will be at real axis
 (c) z will be at unity circle
 (d) None of these

Assertion & Reason Type

Directions : In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :

- (a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.
 (b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1
 (c) Statement-1 is true, Statement-2 is false
 (d) Statement-1 is false, Statement-2 is true

44. **Statement-1 :** Let Z be a complex number satisfying $|Z-3| \leq |Z-1|$, $|Z-3| \leq |Z-5|$, $|Z-i| \leq |Z+i|$, $|Z-i| \leq |Z-5i|$, then area of the region in which Z lies is 12 sq. units.

Statement-2 : Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) $\times$ (distance between parallel sides).

45. z is a unimodular complex number.

Statement-1: $\arg(z^2 + \bar{z}) = \arg z$

Statement-2: $\bar{z} = \cos(\arg z) - i \sin(\arg z)$

46. **Statement-1:** If z_1, z_2 are complex numbers, then $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$.

Statement-2: If 'M' is the midpoint of the line segment PQ, then $OP^2 + OQ^2 = 2OM^2 + 2MP^2$ where 'O' is origin.

47. **Statement-1:** Let $A(z_1), B(z_2)$ and $C(z_3)$ be three points such that $az_1 + bz_2 + cz_3 = 0, a + b + c = 0$ for some $a, b, c \in \mathbb{R}$ (at least one of a, b, c is non-zero). If $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$, then the area of $\triangle OAC$ is $|z_1 z_3|$, O being the origin.

Statement-2: Area of triangle = $\frac{1}{2}(\text{base} \times \text{height})$

48. z_1, z_2 are two distinct points in complex plane such that $2|z_1| = 3|z_2|$ and $z \in C$ be any point.

Statement-1: $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$ is a point such that $-2 \leq \operatorname{Re}(z) \leq 2$.

Statement-2: If $\arg(z_1) = \theta, \arg(z_2) = \theta + \alpha$, then $\frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = 2 \cos \alpha$

Comprehension Type

Paragraph for Q. No. 49 and 50

Consider the set of complex number S defined as

$$S = \left\{ z : \operatorname{Re} \left(\frac{z-1}{z+1} \right) = 0 \right\}$$

49. If $z_1, z_2, z_3 \in S$, then minimum value of $|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2$ is

- (a) 5 (b) 3 (c) 2 (d) 9

50. If z lies on S , then $\arg \left(\frac{2z+i-1}{2z+i+1} \right)$ equals

- (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) $-\pi/4$

Paragraph for Q. No. 51 to 53

$A(z_1), B(z_2), C(z_3)$ are the vertices of a triangle ABC inscribed in the circle $|z| = 2$. Internal angle bisector of the angle A meets the circumcircle again at $D(z_4)$.

51. Complex number representing point D is

- (a) $z_4 = \frac{z_1 z_2}{z_3}$ (b) $z_4 = \frac{z_2 z_3}{z_1}$
(c) $z_4 = \frac{z_1 z_2}{z_3}$ (d) $\sqrt{z_2 z_3}$

52. $\arg \left(\frac{z_4}{z_2 - z_3} \right) =$

- (a) $\pi/4$ (b) $\pi/3$ (c) $\pi/2$ (d) $2\pi/3$

53. $\arg(z_1 z_2)$ is always

- (a) $\arg z_1 + \arg z_2$
(b) $\arg z_1 - \arg z_2$
(c) $\arg z_1 + \arg z_2 + n\pi, n = -1, 0, 1$
(d) $\arg z_1 + \arg z_2 + 2n\pi, n = -1, 0, 1$

Matrix-Match Type

54. z_1, z_2, z_3 are vertices of a triangle. Match the condition in column-I with type of triangle in column-II

Column-I	Column-II
A. $z_1^2 + z_2^2 + z_3^2 = z_2 z_3 + z_3 z_1 + z_1 z_2$	(p) right angled
B. $\operatorname{Re} \left(\frac{z_3 - z_1}{z_3 - z_2} \right) = 0$	(q) obtuse angled
C. $\operatorname{Re} \left(\frac{z_3 - z_1}{z_3 - z_2} \right) < 0$	(r) isosceles and right angled
D. $\frac{z_3 - z_1}{z_3 - z_2} = i$	(s) equilateral

55. Match the following.

Column-I	Column-II
A. If $1, \omega, \omega^2, \dots, \omega^{n-1}$ are the n th roots of unity, then $(2 - \omega)(2 - \omega^2) \dots (2 - \omega^{n-1})$ equals	(p) $2^n - 1$
B. If $z_1, z_2, \dots, z_n$ lie on a circle $ z = 2$, then the value of $ z_1 + z_2 + \dots + z_n - 4 \left \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right $ is	(q) ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots - {}^nC_n$
C. If p is a multiple of n , then sum of the p th power of n th roots of unity is	(r) ${}^nC_{n-1} + {}^nC_{n-2} + {}^nC_{n-3} + \dots + {}^nC_0$
	(s) ${}^nC_1$
	(t) $({}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_n)^{1/2} - 1$

SOLUTIONS

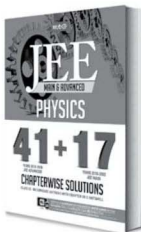
1. (c) $\frac{3+2i \sin \theta}{1-2i \sin \theta}$ will be purely imaginary, if the real

part vanishes, i.e., $\frac{3-4 \sin^2 \theta}{1+4 \sin^2 \theta} = 0$

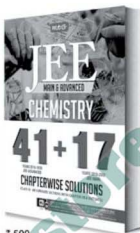
$\Rightarrow 3 - 4 \sin^2 \theta = 0$ (only if θ be real)

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$$\Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2} = \sin \left(\pm \frac{\pi}{3} \right)$$

$$\Rightarrow \theta = n\pi + (-1)^n \left(\pm \frac{\pi}{3} \right) = n\pi \pm \frac{\pi}{3}$$

2. (a): Given, $z = 3 - 4i \Rightarrow (z - 3)^2 = -16$

$$\Rightarrow z^2 - 6z + 25 = 0$$

$$\therefore z^4 - 3z^3 + 3z^2 + 99z - 95$$

$$= (z^2 + 3z - 4)(z^2 - 6z + 25) + 5$$

$$= (z^2 + 3z - 4)(0) + 5 = 5 \quad [\text{From (i)}]$$

3. (a): L.H.S. = $\frac{(\cos x + i \sin x)(\cos y + i \sin y)}{(\cos u + i \sin u)(\cos v + i \sin v)} \sin u \cos v$

$$= \sin u \cos v [\cos(x + y - u - v) + i \sin(x + y - u - v)]$$

4. (b): We have, $(x + iy)^{1/3} = a - ib$

$$\Rightarrow x + iy = (a - ib)^3 = (a^3 - 3ab^2) + i(b^3 - 3a^2b)$$

Equating real and imaginary parts, we get

$$x = a^3 - 3ab^2, y = b^3 - 3a^2b$$

$$\Rightarrow \frac{x}{a} = a^2 - 3b^2, \frac{y}{b} = b^2 - 3a^2$$

Now, $\frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2 = 4(a^2 - b^2) = k(a^2 - b^2)$

$$\therefore k = 4$$

5. (a): Given, that $(a + ib)(c + id)(e + if)(g + ih)$

$$= A + iB$$

$$\Rightarrow (a - ib)(c - id)(e - if)(g - ih) = A - iB$$

Multiplying (i) and (ii), we get

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

6. (b): As given, let $\frac{2z_1}{3z_2} = iy \Rightarrow \frac{z_1}{z_2} = \frac{3}{2}iy$,

$$\therefore \left| \frac{z_1 - z_2}{z_1 + z_2} \right| = \left| \frac{\frac{z_1}{z_2} - 1}{\frac{z_1}{z_2} + 1} \right| = \left| \frac{\frac{3}{2}iy - 1}{\frac{3}{2}iy + 1} \right| = \left| \frac{1 - \frac{3}{2}iy}{1 + \frac{3}{2}iy} \right| = 1 \quad \because |z| = |\bar{z}|$$

7. (b): Let $\frac{z-1}{z+1} = iy$, where $y \in \mathbb{R}$

$$\Rightarrow z = \frac{1+iy}{1-iy} = \frac{1+iy}{1-iy} \times \frac{1+iy}{1+iy} = \frac{(1-y^2) + 2iy}{1+y^2}$$

$$\therefore |z| = \frac{1}{1+y^2} \sqrt{(1-y^2)^2 + 4y^2} = \frac{1+y^2}{1+y^2} = 1$$

8. (c): We have, $|z| - z = 1 + 2i$

$$\text{Let } z = x + iy, \text{ therefore } |x + iy| - (x + iy) = 1 + 2i$$

Equating real and imaginary parts, we get

$$\sqrt{x^2 + y^2} - x = 1 \text{ and } y = -2 \Rightarrow x = \frac{3}{2}$$

Hence complex number $z = \frac{3}{2} - 2i$.

9. (d): We have, $\left| \frac{z_1 - z_2}{z_1 + z_2} \right| = 1 \Rightarrow \frac{z_1 - z_2}{z_1 + z_2} = \cos \alpha + i \sin \alpha$

$$\Rightarrow \frac{2z_1}{-2z_2} = \frac{\cos \alpha + i \sin \alpha + 1}{\cos \alpha - 1 + i \sin \alpha}$$

(Applying componendo and dividendo)

$$\Rightarrow \frac{z_1}{z_2} = i \cot \frac{\alpha}{2} \Rightarrow iz_1 = - \left(\cot \frac{\alpha}{2} \right) z_2$$

But, $iz_1 = kz_2$ [Given] $\Rightarrow k = -\cot \frac{\alpha}{2}$

$$\Rightarrow \cot \frac{\alpha}{2} = -k \Rightarrow \tan \alpha = \frac{-2k}{k^2 - 1}$$

$$\Rightarrow \tan \alpha = \frac{2k}{1 - k^2} \Rightarrow \alpha = \tan^{-1} \left(\frac{2k}{1 - k^2} \right) = 2 \tan^{-1} k$$

Now, $\frac{z_1 - z_2}{z_1 + z_2} = \cos \alpha + i \sin \alpha$

$$\Rightarrow \alpha \text{ is the angle between } z_1 - z_2 \text{ and } z_1 + z_2.$$

10. (c): $\left| \frac{1}{2}(z_1 + z_2) + \sqrt{z_1 z_2} \right| + \left| \frac{1}{2}(z_1 + z_2) - \sqrt{z_1 z_2} \right|$

$$= \frac{1}{2} |(\sqrt{z_1} + \sqrt{z_2})^2| + \frac{1}{2} |(\sqrt{z_1} - \sqrt{z_2})^2|$$

$$= \frac{1}{2} |\sqrt{z_1} + \sqrt{z_2}|^2 + \frac{1}{2} |\sqrt{z_1} - \sqrt{z_2}|^2 \quad (\because |z|^2 = |z|^2)$$

$$= \frac{1}{2} 2[|\sqrt{z_1}|^2 + |\sqrt{z_2}|^2] = |z_1| + |z_2|$$

11. (a): Here, $|z| = 1 \Rightarrow |x + iy| = 1 \Rightarrow x^2 + y^2 = 1$

$$\omega = \frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x^2 + y^2 - 1) + 2iy}{(x+1)^2 + y^2} = \frac{2iy}{(x+1)^2 + y^2}$$

$$\therefore \operatorname{Re}(\omega) = 0$$

12. (c): As given, $\alpha - i\beta = \frac{3-4xi}{3+4xi}$. Taking modulus and

squaring on both sides, we get $\alpha^2 + \beta^2 = 1$.

13. (c): As given, $(\sqrt{8} + i)^{50} = 3^{49}(a + ib)$

Taking modulus and squaring on both sides, we get

$$(8 + 1)^{50} = 3^{98}(a^2 + b^2) \Rightarrow 9^{50} = 3^{98}(a^2 + b^2)$$

$$\Rightarrow 3^{100} = 3^{98}(a^2 + b^2) \Rightarrow (a^2 + b^2) = 9$$

14. (d): Since $1 - i = \sqrt{2} \left\{ \cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right\} \Rightarrow |1 - i| = \sqrt{2}$

$$\therefore |1 - i|^x = 2^x \Rightarrow (\sqrt{2})^x = 2^x \Rightarrow 2^{x/2} = 2^x \Rightarrow x = 0$$

Therefore, the number of non-zero integral solutions is nil or zero.

$$15. (c) : 1 + i\sqrt{3} = 2 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 2e^{i\pi/3}$$

$$\therefore (1 + i\sqrt{3})^9 = (2e^{i\pi/3})^9 = 2^9 \cdot e^{i(3\pi)} = 2^9 (\cos 3\pi + i \sin 3\pi) = -2^9$$

$$\therefore a + ib = (1 + i\sqrt{3})^9 = -2^9 \quad \therefore b = 0$$

$$16. (d) : \text{We have, } x + \frac{1}{x} = \sqrt{3}$$

$$\Rightarrow x^2 - \sqrt{3}x + 1 = 0 \Rightarrow x = \frac{\sqrt{3} \pm \sqrt{3-4}}{2}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm i}{2} = \frac{\sqrt{3}}{2} \pm \frac{i}{2}$$

$$\Rightarrow x = \cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \quad [\text{Taking +ve sign}]$$

17. (a) : It is given that $OP = OQ$

$$\therefore |OP| = |OQ| \Rightarrow |a + ib| = |c + id|$$

$$18. (c) : \text{We have, } z_k = 1 + a + a^2 + \dots + a^{k-1} = \frac{1-a^k}{1-a}$$

$$\Rightarrow z_k - \frac{1}{1-a} = \frac{-a^k}{1-a}$$

$$\Rightarrow \left| z_k - \frac{1}{1-a} \right| = \frac{|a^k|}{|1-a|} = \frac{|a|^k}{|1-a|} < \frac{1}{|1-a|}$$

$$\Rightarrow z_k \text{ lies within } \left| z - \frac{1}{1-a} \right| = \frac{1}{|1-a|}$$

$$19. (d) : \text{Given, } \frac{|z-2|}{|z-3|} = 2$$

$$\Rightarrow \sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$$

$$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$$

$$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0 \quad \dots(i)$$

We know that, standard equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \dots(ii)$$

Comparing (i) and (ii), we get

$$2g = -\frac{20}{3} \Rightarrow g = -\frac{10}{3}, f = 0, c = \frac{32}{3}$$

$$\text{Hence, Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

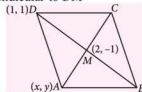
20. (a) : We have, $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$
Therefore the point having affix z is equidistant from the four points having affixes z_1, z_2, z_3, z_4 . Thus z is the affix of either the centre of a circle or the point of intersection of diagonals of a square or rectangle. Therefore z_1, z_2, z_3, z_4 are either concyclic or vertices of a square. Hence z_1, z_2, z_3, z_4 are concyclic.

21. (a) : We have, $BD = 2AC \Rightarrow 2DM = 2(2AM)$

$$\Rightarrow DM = 2AM \Rightarrow DM^2 = 4AM^2$$

$$\Rightarrow 5 = 4[(x-2)^2 + (y+1)^2] \quad \dots(i)$$

Again slope of $DM = -2$ and slope of AM is $\frac{y+1}{x-2}$
 AM is perpendicular to DM



$$\therefore -2 \left(\frac{y+1}{x-2} \right) = -1 \Rightarrow x-2 = 2(y+1) \quad \dots(ii)$$

Hence from (i) and (ii), we have

$$y = -\frac{1}{2}, -\frac{3}{2} \text{ and } x = 3, 1$$

$$22. (d) : \text{Here, } z_2 = z_1 e^{2i\pi/3} = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$= \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = -\frac{3}{4} - \frac{1}{4}i$$

23. (b) : We have, $|z-2| + |z+2| = 8$

$$\Rightarrow \sqrt{(x-2)^2 + y^2} + \sqrt{(x+2)^2 + y^2} = 8$$

$$\Rightarrow -8x - 64 = -16\sqrt{(x+2)^2 + y^2}$$

$$\Rightarrow x+8 = 2\sqrt{(x+2)^2 + y^2}$$

$$\Rightarrow 3x^2 + 4y^2 - 48 = 0 \Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1,$$

which is an ellipse.

24. (b) : Let $z_1 = 1 + 3i, z_2 = 5 + i$ and $z_3 = 3 + 2i$

Then area of triangle formed by z_1, z_2 and z_3

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 5 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

Hence, z_1, z_2 and z_3 are collinear.

25. (b) : Here, $z = x + iy$, then $z + iz = (x-y) + i(x+y)$ and $iz = -y + ix$

If A denotes the area of the triangle formed by $z,$

$$z + iz \text{ and } iz, \text{ then } A = \frac{1}{2} \begin{vmatrix} x & y & 1 \\ x-y & x+y & 1 \\ -y & x & 1 \end{vmatrix}$$

Applying transformation $R_2 \rightarrow R_2 - R_1 - R_3$, we get

$$A = \frac{1}{2} \begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 1 \end{vmatrix} = \frac{1}{2} (x^2 + y^2) = \frac{1}{2} |z|^2$$

26. (a) : Given points are $A(3 + 4i)$, $B(5 - 2i)$ and $C(-1 + 16i)$.

$$\therefore \text{Area of triangle} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 3 & 4 & 1 \\ 5 & -2 & 1 \\ -1 & 16 & 1 \end{vmatrix} = 0$$

Hence, points are collinear.

27. (d) : The affix of G is $\frac{z_1 + z_2 + z_3}{3}$. Since $z = 0$ is the mid point of AG . Therefore affix of the mid-point of AG is 0.

$$\Rightarrow \frac{\frac{z_1 + z_2 + z_3}{3} + z_1}{1+1} = 0 \Rightarrow 4z_1 + z_2 + z_3 = 0$$

28. (d) : Given, $z_1 = 1 + i \Rightarrow z_1 = (1, 1)$,

$$z_2 = -2 + 3i \Rightarrow z_2 = (-2, 3),$$

$$z_3 = \frac{ai}{3} \Rightarrow z_3 = (0, a/3)$$

Since, z_1, z_2 and z_3 are collinear

[Given]

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ -2 & 3 & 1 \\ 0 & a/3 & 1 \end{vmatrix} = 0 \Rightarrow -\frac{a}{3}(1+2)+1(3+2)=0$$

$$\Rightarrow -a + 5 = 0 \Rightarrow a = 5$$

29. (b) : Given vertices are $0 = 0 + i0$, $z = x + iy$ and $ze^{i\alpha} = (x + iy)(\cos\alpha + i\sin\alpha)$

$$\therefore \text{Area} = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ (x \cos \alpha - y \sin \alpha) & (y \cos \alpha + x \sin \alpha) & 1 \end{vmatrix}$$

$$= \frac{1}{2} [xy \cos \alpha + x^2 \sin \alpha - xy \cos \alpha + y^2 \sin \alpha]$$

$$= \frac{1}{2} \sin \alpha (x^2 + y^2) = \frac{1}{2} |z|^2 \sin \alpha \quad [\because |z| = \sqrt{x^2 + y^2}]$$

30. (a) : Given that $\left(\frac{z+i}{z+2}\right)$ is purely imaginary.

$$\text{Let } z = x + iy \Rightarrow \frac{x+iy+i}{x+iy+2} = \frac{x+i(y+1)}{(x+2)+iy}$$

$$= \left[\frac{x^2 + 2x + y^2 + y}{(x+2)^2 + y^2} \right] + i \left[\frac{(y+1)(x+2) - xy}{(x+2)^2 + y^2} \right]$$

Since, it is purely imaginary, then real part must be equal to zero.

$$\Rightarrow \frac{x^2 + y^2 + 2x + y}{(x+2)^2 + y^2} = 0 \Rightarrow x^2 + y^2 + 2x + y = 0,$$

which is a circle and its radius is given by

$$\sqrt{1 + \frac{1}{4} - 0} = \frac{\sqrt{5}}{2}$$

31. (b) : Given, $|z+1| = \sqrt{2}|z-1|$... (i)

$$\text{Put } z = x + iy \text{ in (i)} \Rightarrow |x+iy+1| = \sqrt{2}|x+iy-1|$$

$$\Rightarrow |(x+1)+iy| = \sqrt{2}|(x-1)+iy|$$

$$\Rightarrow x^2 + y^2 - 6x + 1 = 0, \text{ which is the equation of a circle.}$$

32. (b) : We have $\left|\frac{z-a}{z+\bar{a}}\right| = 1$

$$\Rightarrow |z-a| = |z+\bar{a}| \Rightarrow |z-a|^2 = |z+\bar{a}|^2$$

$$\Rightarrow (z-a)(\bar{z}-\bar{a}) = (z+\bar{a})(\bar{z}+\bar{a})$$

$$\Rightarrow (z-a)(\bar{z}-\bar{a}) = (z+\bar{a})(\bar{z}+\bar{a})$$

$$\Rightarrow z\bar{z} - z\bar{a} - a\bar{z} + a\bar{a} = z\bar{z} + z\bar{a} + a\bar{z} + a\bar{a}$$

$$\Rightarrow z\bar{a} + z\bar{a} + \bar{a}\bar{z} + a\bar{z} = 0 \Rightarrow (a+\bar{a})(z+\bar{z}) = 0$$

$$\Rightarrow z + \bar{z} = 0 \quad (\because a + \bar{a} = 2\text{Re}(a) \neq 0)$$

$$\Rightarrow 2\text{Re}(z) = 0 \Rightarrow 2x = 0 \Rightarrow x = 0,$$

which is the equation of y -axis.

33. (c) : We have $|z-1| + |z+1| \leq 4$

$$\Rightarrow |z-1|^2 + |z+1|^2 + 2|z-1||z+1| \leq 16$$

$$\Rightarrow (z-1)(\bar{z}-1) + (z+1)(\bar{z}+1) + 2|(z-1)(z+1)| \leq 16$$

$$\Rightarrow 2|z|^2 + 2+2|z^2-1| \leq 16 \Rightarrow |z|^2 + |z^2-1| \leq 7$$

$$\Rightarrow |x+iy|^2 + |(x+iy)^2-1| \leq 7 \Rightarrow \frac{x^2}{4} + \frac{y^2}{3} \leq 1 \text{ (ellipse)}$$

Therefore the point z is on the boundary or in the interior of the ellipse.

$$34. (b) : \text{We have } \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$$

$$\therefore \arg \frac{z-1}{z+1} = \tan^{-1} \frac{2y}{x^2+y^2-1}$$

$$\Rightarrow \tan^{-1} \frac{2y}{x^2+y^2-1} = \frac{\pi}{3} \quad [\because \arg \left(\frac{z-1}{z+1} \right) = \frac{\pi}{3} \text{ (Given)}]$$

$$\Rightarrow \frac{2y}{x^2+y^2-1} = \tan \frac{\pi}{3} = \sqrt{3}$$

$$\Rightarrow x^2 + y^2 - 1 = \frac{2}{\sqrt{3}} y \Rightarrow x^2 + y^2 - \frac{2}{\sqrt{3}} y - 1 = 0,$$

which is obviously a circle.

$$35. (b) : \text{We have } \frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{2(x+1)+2iy}{(1-y)+ix}$$

$$= \frac{[(2x+1)(1-y)+2xy] + i[2y(1-y)-x(2x+1)]}{(1-y)^2 + x^2}$$

Also, imaginary part of $\frac{(2z+1)}{(iz+1)} = -2$ [Given]

$\therefore x + 2y - 2 = 0$, which is a straight line.

36. (b): Let $z = (x + iy) \Rightarrow z^2 = x^2 - y^2 + 2ixy$
Given, $\operatorname{Re}(z^2) = 1 \Rightarrow x^2 - y^2 = 1$, which is a hyperbola.

37. (c): We have, $|z - 1| = |z + i|$

$$\Rightarrow |x - 1 + iy|^2 = |x + i(y + 1)|^2$$

$$\Rightarrow (x - 1)^2 + y^2 = x^2 + (y + 1)^2$$

$\Rightarrow x + y = 0$ i.e., a straight line through the origin.

38. (a): Given, $|z - 2 + i| = |z - 3 - i|$

$$\Rightarrow |(x - 2) + i(y + 1)| = |(x - 3) + i(y - 1)|$$

$$\Rightarrow \sqrt{(x - 2)^2 + (y + 1)^2} = \sqrt{(x - 3)^2 + (y - 1)^2}$$

$$\Rightarrow x^2 + 4 - 4x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 1 - 2y$$

$$\Rightarrow 2x + 4y - 5 = 0.$$

39. (a): Given, $|iz - 1| + |z - i| = 2$

$$\Rightarrow |i(x + iy) - 1| + |x + iy - i| = 2$$

$$\Rightarrow |-(y + 1) + ix| + |x + i(y - 1)| = 2$$

$$\Rightarrow \sqrt{-(y + 1)^2 + x^2} + \sqrt{x^2 + (y - 1)^2} = 2$$

$$\Rightarrow \sqrt{(y + 1)^2 + x^2} = 2 - \sqrt{x^2 + (y - 1)^2}$$

$$\Rightarrow 4y = 4 - 4\sqrt{x^2 + (y - 1)^2} \Rightarrow y = 1 - \sqrt{x^2 + (y - 1)^2}$$

$$\Rightarrow x^2 + (y - 1)^2 = (1 - y)^2$$

$$\Rightarrow x^2 = 0 \Rightarrow x = 0 \text{ i.e., equation of straight line.}$$

40. (d): Given, $\left|z + \frac{i}{2}\right| = \left|z - \frac{i}{2}\right|$

$$\Rightarrow \left|x + iy + \frac{i}{2}\right|^2 = \left|x + iy - \frac{i}{2}\right|^2$$

$$\Rightarrow \left|x + i\left(y + \frac{1}{2}\right)\right|^2 = \left|x + i\left(y - \frac{1}{2}\right)\right|^2$$

$$\Rightarrow x^2 + \left(y + \frac{1}{2}\right)^2 = x^2 + \left(y - \frac{1}{2}\right)^2$$

$$\Rightarrow 2y = 0 \Rightarrow y = 0 \text{ i.e., } x\text{-axis.}$$

41. (d): $z - 2 - 3i = x + iy - 2 - 3i = (x - 2) + i(y - 3)$

$$\therefore \tan^{-1}\left(\frac{y - 3}{x - 2}\right) = \frac{\pi}{4} \Rightarrow \frac{y - 3}{x - 2} = \tan \frac{\pi}{4} = 1$$

$$\Rightarrow x - y + 1 = 0$$

42. (b): We have $|z^2 - 1| = |z|^2 + 1$

$$\Rightarrow |(x + iy)^2 - 1| = |x + iy|^2 + 1$$

$$\Rightarrow |(x^2 - y^2 - 1) + 2xyi| = \left(\sqrt{x^2 + y^2}\right)^2 + 1$$

$$\Rightarrow x^4 + y^4 + 1 - 2x^2y^2 + 2y^2 - 2x^2 + 4x^2y^2 = x^4 + y^4 + 1 + 2x^2y^2 + 2y^2 + 2x^2$$

$$\Rightarrow 2x^2y^2 = 2x^2y^2 + 4x^2 \Rightarrow x = 0$$

Now, $z = x + iy = 0 + iy = iy$

Hence z lies on imaginary axis.

43. (b): Given, $\omega = \frac{1 - iz}{z - i}$, then $|\omega| = 1$

$$\Rightarrow \left|\frac{1 - iz}{z - i}\right| = 1 \Rightarrow |1 - iz| = |z - i|$$

$$\Rightarrow |(1 + y) - ix| = |x + i(y - 1)|$$

$$\Rightarrow \sqrt{x^2 + 1 + y^2 + 2y} = \sqrt{x^2 + y^2 + 1 - 2y} \Rightarrow y = 0$$

Hence, $z = x + iy = x$. So z lies on real axis.

44. (d): Let $Z = x + iy$

We know that

$$Z + \bar{Z} = 2\operatorname{Re}(Z) = 2x, Z - \bar{Z} = 2i\operatorname{Im}(Z) = 2iy$$

$$\therefore |Z - 3| \leq |Z - 1| \Rightarrow |Z - 3|^2 \leq |Z - 1|^2$$

$$\Rightarrow (Z - 3)(\bar{Z} - 3) \leq (Z - 1)(\bar{Z} - 1)$$

$$\Rightarrow Z\bar{Z} - 3(Z + \bar{Z}) + 9 \leq Z\bar{Z} - 1(Z + \bar{Z}) + 1$$

$$\Rightarrow 2(Z + \bar{Z}) \geq 8 \Rightarrow x \geq 2$$

Similarly, $|Z - 3| \leq |Z - 5| \Rightarrow x \leq 4$

$$|Z - i| \leq |Z + i| \Rightarrow (Z - i)(\bar{Z} + i) \leq (Z + i)(\bar{Z} - i)$$

$$\Rightarrow Z\bar{Z} + i(Z - \bar{Z}) + 1 \leq Z\bar{Z} + i(\bar{Z} - Z) + 1$$

$$\Rightarrow 2i(Z - \bar{Z}) \leq 0 \Rightarrow 2i(2iy) \leq 0 \Rightarrow -4y \leq 0 \Rightarrow y \geq 0$$

Similarly, $|Z - i| \leq |Z - 5i| \Rightarrow y \leq 3$

Hence, $2 \leq x \leq 4$ and $0 \leq y \leq 3$

They form rectangle and its area = 6 sq. units

45. (d): z is a unimodular complex number

$$\Rightarrow |z| = 1 \Rightarrow z = \cos\theta + i\sin\theta, \text{ where } \theta = \arg(z)$$

$$\therefore \bar{z} = \cos\theta - i\sin\theta$$

So, statement-2 is true

$$\text{Now, } \arg(z^2 + \bar{z}) = \arg(\cos 2\theta + i\sin 2\theta + \cos\theta - i\sin\theta)$$

$$= \tan^{-1} \frac{\sin 2\theta - \sin\theta}{\cos 2\theta + \cos\theta} = \tan^{-1} \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\cos\frac{\theta}{2}}$$

$$= \tan^{-1} \tan \frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2} \arg z$$

46. (a): Let $OM = z_1$,

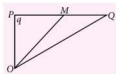
$$MP = z_2, MQ = -z_2$$

Since,

$$OP^2 + OQ^2 = 2MP^2 + 2OM^2$$

$$\therefore |z_1 + z_2|^2 + |z_1 - z_2|^2 =$$

$$2|z_1|^2 + 2|z_2|^2$$



47. (d): Let $c \neq 0$,

Now $a + b + c = 0$

$$\Rightarrow c = -(a+b) \Rightarrow z_3 = \frac{az_1 + bz_2}{a+b}$$

$\Rightarrow z_1, z_2, z_3$ are collinear

$$\text{Area of } \triangle OAC = \frac{1}{2} OA \times OC = \frac{1}{2} |z_1| |z_3|$$

Taking B as centre and AB as radius draw a circle. It will pass through $O \Rightarrow OAC$ is right angled triangle right angled at O

$$\begin{aligned} 48. (a): z &= \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} \\ &= \frac{2|z_1| \operatorname{cis} \theta}{3|z_2| \operatorname{cis}(\theta + \alpha)} + \frac{3|z_2| \operatorname{cis}(\theta + \alpha)}{2|z_1| \operatorname{cis} \theta} \\ &= \operatorname{cis}(-\alpha) + \operatorname{cis} \alpha = 2\cos \alpha \in [-2, 2] \end{aligned}$$

49. (b): Clearly, S represents the set of complex number lying on the circle $|z| = 1, z \neq -1$.

$$\begin{aligned} \therefore |z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2 \\ = 3 + (z_1 + z_2 + z_3)(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) \\ = 3 + |z_1 + z_2 + z_3|^2 \geq 3. \end{aligned}$$

50. (b): From the diagram

$$\begin{aligned} \arg \left(\frac{\frac{1}{2} - \frac{i}{2} - z}{\frac{1}{2} - \frac{i}{2} - z} \right) &= \frac{\pi}{4} \\ \Rightarrow \arg \left(\frac{2z + i - 1}{2z + i + 1} \right) &= \frac{\pi}{4} \end{aligned}$$



51. (d)

52. (c)

53. (d)

$\frac{z_4}{z_2} = e^{iA}, \frac{z_3}{z_4} = e^{iA}$ and OD is the angular bisector of

$\angle BOC$ is isosceles

$\therefore OD \perp BC$

$$\therefore \arg \left(\frac{z_4}{z_2 - z_3} \right) = \frac{\pi}{2}.$$

54. $A \rightarrow (s); B \rightarrow (p); C \rightarrow (q); D \rightarrow (r)$

$$(A) \frac{z_3 - z_1}{z_2 - z_1} = \frac{z_1 - z_2}{z_3 - z_2} = e^{i\frac{\pi}{3}}$$

$$(B) \operatorname{Re} \left(\frac{z_3 - z_1}{z_3 - z_2} \right) = 0 \Rightarrow \arg \left(\frac{z_3 - z_1}{z_3 - z_2} \right) = \pm \frac{\pi}{2}$$

$$(C) \operatorname{Re} \left(\frac{z_3 - z_1}{z_3 - z_2} \right) < 0 \Rightarrow \arg \left(\frac{z_3 - z_1}{z_3 - z_2} \right) > \frac{\pi}{2}$$

$$(D) \left(\frac{z_3 - z_1}{z_3 - z_2} \right) = i = e^{i\frac{\pi}{2}} \text{ and } |z_3 - z_1| = |z_3 - z_2|$$

55. $A \rightarrow (p, r, t); B \rightarrow (q); C \rightarrow (s)$

$$(A) (z^n - 1) = (z - 1)(z - \omega) \dots (z - \omega^{n-1})$$

Put $z = 2$

$$\begin{aligned} (B) 4 \left| \frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n} \right| &= \left| \frac{|z_1|^2}{z_1} + \frac{|z_2|^2}{z_2} + \dots + \frac{|z_n|^2}{z_n} \right| \\ &= |z_1 + z_2 + \dots + z_n| = |z_1 + z_2 + \dots + z_n| \end{aligned}$$

Hence, answer is 0.

(C) Since p is the multiple of n , all will be 1 and sum = n

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CLASS XI Series 7

CBSE DRILL



Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Straight Lines / Conic Sections / Introduction to Three Dimensional Geometry

Straight Lines

DEFINITION

A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

Distance Formula		Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Section Formula	Internal Division	Coordinates of M , if M divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$.
	External Division	Coordinates of M , if M divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m : n$ is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$.
Mid Point Formula		Coordinates of M , if M bisects the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.
Area of a Triangle		Area of $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is $\Delta = \frac{1}{2} x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) $.

SLOPE OF A LINE

The trigonometrical tangent of the angle that a line makes with the positive direction of the x -axis in anticlockwise sense is called the slope or gradient of the line, denoted by m .

Note :

- Slope of a line parallel to x -axis is 0.
- Slope of a line perpendicular to x -axis is not defined.
- Slope of a line passing through two points (x_1, y_1) and $(x_2, y_2) = (y_2 - y_1) / (x_2 - x_1)$.

CONDITION OF PARALLELISM AND PERPENDICULARITY

Two lines having slopes m_1 and m_2 are said to be

- Parallel iff $m_1 = m_2$
- Perpendicular iff $m_1 m_2 = -1$

ANGLE BETWEEN TWO LINES

Angle between two lines having slopes m_1 and m_2 is

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|, 1 + m_1 m_2 \neq 0.$$

COLLINEARITY OF THREE POINTS

Three points A, B and C in a plane are said to be collinear, if slope of $AB =$ slope of BC .

DIFFERENT FORMS OF EQUATION OF LINE

- Equation of x-axis :** Equation of the x-axis is $y = 0$.
- Equation of y-axis :** Equation of the y-axis is $x = 0$.
- Equation of line parallel to x-axis :**

Equation of a line parallel to x-axis is of the form $y = c$.

- Equation of line parallel to y-axis :** Equation of a line parallel to y-axis is of the form $x = d$.

- Slope intercept form :**

Equation of line having slope m and cuts an intercept c on y-axis is $y = mx + c$.

- Point slope form :** Equation of line whose slope is m and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

- Two point form :** Equation of line passing through

$$A(x_1, y_1) \text{ and } B(x_2, y_2) \text{ is } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

- Intercept form :** Equation of line cuts intercepts a and b on x and y-axes respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

- Normal form :** Equation of line having normal distance from origin p and this normal makes an angle α with the +ve x-axis is $x \cos \alpha + y \sin \alpha = p$.

GENERAL EQUATION OF A LINE

An equation of first degree in x and y is $ax + by + c = 0$, which represents an equation of line. Slope of this line is

$$-\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } y)} = -\frac{a}{b}$$

- Equation of line can be written as $y = -\frac{ax}{b} - \frac{c}{b}$
- The intercept form of a line is $\frac{x}{-c/a} + \frac{y}{-c/b} = 1$.

Distance of a Point from a Line

Distance of a point (x_1, y_1) from a line $ax + by + c = 0$ is,

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Distance between Two Parallel Lines

Distance between two parallel lines $ax + by + c_1 = 0$

$$\text{and } ax + by + c_2 = 0 \text{ is, } d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}.$$

FAMILY OF LINES

If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are two non-parallel lines, then $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$, represents the family of all those lines, which passes through the point of intersection of given lines.

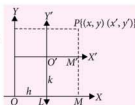
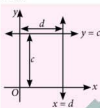
SHIFTING OF ORIGIN

Translation of axes

An equation corresponding to a set of points with reference to a system of coordinate axes may be simplified by taking the set of points in some other suitable coordinate system such that all geometric properties remain unchanged. We can form a transformation in which the new axes can be transformed parallel to the original axes and origin can be shifted to a new point. This kind of transformation is called a translation of axes.

The coordinates of each point of the plane are changed under a translation of axes. To see how coordinates of a point of the plane changed under a translation of axes, let us take a point $P(x, y)$ referred to the axes OX and OY . Let $O'X'$ and $O'Y'$ be new axes parallel to OX and OY respectively, where O' is the new origin. Let (h, k) be the coordinates of O' referred to the old axes, i.e., $OL = h$ and $LO' = k$. Also, $OM = x$ and $MP = y$. The transformation relation between new coordinates (x', y') and old coordinates (x, y) are given by

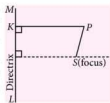
$$x = x' + h, y = y' + k$$



Conic Sections

DEFINITION

The locus of a point which moves in a plane such that the ratio of its distance from a fixed point to its perpendicular distance from a fixed straight line is always constant, is known as a conic section or a conic. The fixed point is called focus of the conic, fixed line is called directrix of the conic and the constant ratio is called eccentricity of the conic. From figure, we have



$$\frac{PS}{PK} = \text{constant} = e$$

- If $e = 0$, then the curve is a circle.
- If $e = 1$, then the curve is a parabola.
- If $e < 1$, then the curve is an ellipse.
- If $e > 1$, then the curve is a hyperbola.

CIRCLE

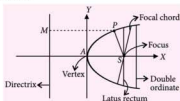
A circle is defined as the locus of all such points in a plane, which remains at a constant distance from a fixed point. Here the fixed point is called the centre of the circle and the constant distance is called its radius.

EQUATION OF CIRCLE

- **Standard form** : The equation of a circle with centre (h, k) and radius r is given by $(x - h)^2 + (y - k)^2 = r^2$.
- **General form** : The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, where centre $= (-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$.

PARABOLA

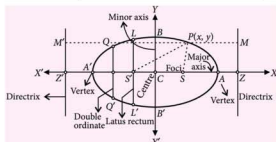
A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point. The fixed line is called the directrix and fixed point is called focus of the parabola.

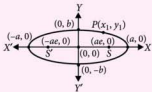
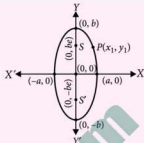


Fundamental terms	Horizontal parabola	Vertical parabola
Standard equation	$y^2 = 4ax$	$x^2 = 4ay$
Graph		
Eccentricity	$e = 1$	$e = 1$
Coordinates of focus	$(a, 0)$	$(0, a)$
Equation of directrix	$x + a = 0$	$y + a = 0$
Equation of axis	$y = 0$	$x = 0$
Coordinates of vertex	$(0, 0)$	$(0, 0)$
Extremities of latus rectum	$(a, \pm 2a)$	$(\pm 2a, a)$
Length of latus rectum	$4a$	$4a$
Equation of latus rectum	$x - a = 0$	$y - a = 0$

ELLIPSE

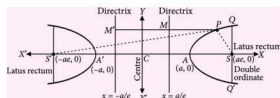
An ellipse is the set of all points in a plane, the sum of whose distance from two fixed points in the plane is constant. The two fixed points are called the foci of the ellipse.



Fundamental terms	Horizontal ellipse	Vertical ellipse
Standard equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a < b$
Graph		
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	(±a, 0)	(0, ±b)
Length of major axis	2a	2b
Length of minor axis	2b	2a
Coordinates of foci	(±ae, 0)	(0, ±be)
Equation of directrices	$x = \pm \left(\frac{a}{e} \right)$	$y = \pm \left(\frac{b}{e} \right)$
Eccentricity	$e = \sqrt{1 - \frac{b^2}{a^2}}$	$e = \sqrt{1 - \frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Ends of latus rectum	$\left(\pm ae, \pm \frac{b^2}{a} \right)$	$\left(\pm \frac{a^2}{b}, \pm be \right)$
Focal distance or radii	$ SP = (a - ex_1)$ and $ S'P = (a + ex_1)$	$ SP = (b - ey_1)$ and $ S'P = (b + ey_1)$
Sum of focal radii = $ SP + S'P $	2a	2b
Distance between foci	2ae	2be

HYPERBOLA

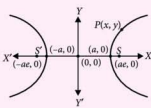
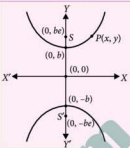
A hyperbola is the set of all points in a plane, the difference of whose distance from two fixed points is constant.



MPP-7 CLASS XI

ANSWER KEY

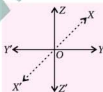
- | | | | |
|-------------|-----------|-------------|-------------|
| 1. (c) | 2. (b) | 3. (b) | 4. (c) |
| 5. (c) | 6. (b) | 7. (a,c) | 8. (a,b,c) |
| 9. (a,b,c) | 10. (b,c) | 11. (a,b,c) | 12. (a,b,c) |
| 13. (a,b,d) | 14. (b) | 15. (c) | 16. (c) |
| 17. (10) | 18. (5) | 19. (3) | 20. (1) |

Fundamental terms	Hyperbola	Conjugate Hyperbola
Standard equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Graph		
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	(±a, 0)	(0, ±b)
Length of transverse axis	2a	2b
Length of conjugate axis	2b	2a
Coordinates of foci	(±ae, 0)	(0, ±be)
Equation of directrices	$x = \pm \left(\frac{a}{e} \right)$	$y = \pm \left(\frac{b}{e} \right)$
Eccentricity	$e = \sqrt{1 + \frac{b^2}{a^2}}$	$e = \sqrt{1 + \frac{a^2}{b^2}}$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Ends of latus rectum	$\left(\pm ae, \pm \frac{b^2}{a} \right)$	$\left(\pm \frac{a^2}{b}, \pm be \right)$
Difference of focal radii = SP - S'P	2a	2b
Distance between foci	2ae	2be

Introduction To Three Dimensional Geometry

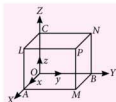
COORDINATE AXES AND COORDINATE PLANE

Let XOX' , YOY' and ZOZ' be three mutually perpendicular lines intersecting at O . The lines XOX' , YOY' and ZOZ' are called coordinate axes. Also the planes XOY , YOZ and ZOX are known as coordinate planes.



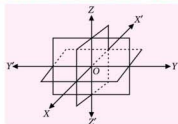
COORDINATES OF A POINT

- Let P be a point in space and $OA = x$, $OB = y$ and $OC = z$. Then the coordinates of P are (x, y, z) .



- Coordinates of any point on x -axis is $(x, 0, 0)$, on y -axis is $(0, y, 0)$ and on z -axis is $(0, 0, z)$.
- Coordinates of any point in yz -plane be $(0, y, z)$, in xy -plane be $(x, y, 0)$ and in zx -plane be $(x, 0, z)$. O is the origin and coordinates of O are $(0, 0, 0)$.

SIGNS OF COORDINATES IN EIGHT OCTANTS



The octants $XOYZ$, $X'OYZ$, $X'OY'Z$, $XOY'Z$, $XOYZ'$, $X'OYZ'$ and $XOY'Z'$ are denoted by I, II, III, ..., VIII respectively.

	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

Distance Formula		Distance between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
Section Formula	Internally	Coordinates of $M(x, y, z)$, if M divides the join of $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ internally in the ratio $m : n$ is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$.
	Externally	Coordinates of $M(x, y, z)$, if M divides the join of $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ externally in the ratio $m : n$ is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$.
Mid point Formula		Coordinates of $M(x, y, z)$, if M is the mid point of line segment joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$.
Centroid of a Triangle		Coordinates of $G(x, y, z)$, if G is the centroid of triangle, whose vertices are (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$.

WORK IT OUT**VERY SHORT ANSWER TYPE**

- If two vertices of a parallelogram are $(3, 2)$, $(-1, 0)$ and its diagonals meet at $(2, -5)$, find the other two vertices of the parallelogram.
- Find the transformed equation of the curve $xy - y^2 - x + y = 0$ when the origin is shifted to point $(1, 1)$.
- Find the equation of a concentric circle with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and of double its area.
- If the eccentricity of an ellipse is $\frac{5}{8}$ and the distance between its foci is 10, then find the latus rectum of the ellipse.
- Find the values of a if the distance between points $P(a, -8, 4)$ and $Q(-3, -5, 4)$ is 5.

SHORT ANSWER TYPE

- A straight line cuts intercepts from the axes of coordinates the sum of whose reciprocals is a constant. Show that it always passes through a fixed point.
- If the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$, prove that $ln = am^2$.

- Show that the coplanar points $(0, 4, 1)$, $(2, 3, -1)$, $(4, 5, 0)$ and $(2, 6, 2)$ are the vertices of a square.
- Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.
- If the latus rectum of an ellipse is equal to half of minor axis, find its eccentricity.

LONG ANSWER TYPE - I

- The line $2x - y = 5$ turns about the point on it, where the ordinate is equal to the abscissa through an angle of 45° in the anticlockwise direction. Find the equation of the line in the new position.
- Prove that the line $5x - 2y - 1 = 0$ is mid-parallel to the lines $5x - 2y - 9 = 0$ and $5x - 2y + 7 = 0$.
- Find the equation of the hyperbola with centre at origin, transverse axis along x -axis, eccentricity $\sqrt{5}$ and sum of whose semi-axes is 9.
- A ray of light passing through the point $P(1, 2)$ reflects on the x -axis at the point A and the reflected ray passes through the point $Q(5, 3)$. Find the coordinates of A .
- Find the equations of the circles touching y -axis at $(0, 3)$ and making intercept of 8 units on the x -axis.

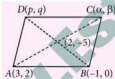
LONG ANSWER TYPE - II

16. Show that the locus of the mid-point of the distance between the axes of the variable line $x \cos \alpha + y \sin \alpha = p$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$, where p is a constant.
17. The mid-points of the sides of a triangle are $(5, 7, 11)$, $(0, 8, 5)$ and $(2, 3, -1)$. Find the coordinates of the vertices of the triangle.
18. An arch way is in the shape of a semi ellipse, the road level being the major axis. If the breadth of the arch way is 30 feet and a man 6 feet tall just touches the top when 2 feet from the side, find the greatest height of the arch.
19. If P and Q are the end points of a focal chord of the parabola $y^2 = 4ax$ with focus F , then prove that $\frac{1}{|FP|} + \frac{1}{|FQ|} = \frac{2}{l}$, where l is the length of semi-latus rectum of the parabola.
20. A line is such that its segment between the straight lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Obtain its equation.

SOLUTIONS

1. Mid-point of the line segment joining given vertices $(3, 2)$ and $(-1, 0)$ is $(1, 1)$, which is different from the point $(2, -5)$, therefore, the given vertices cannot be opposite vertices.

Let A, B be the given vertices $(3, 2)$, $(-1, 0)$ and let C, D be the vertices (α, β) , (p, q) respectively. As diagonals meet at $(2, -5)$



$$\therefore \frac{\alpha+3}{2} = 2, \frac{\beta+2}{2} = -5; \frac{p-1}{2} = 2, \frac{q+0}{2} = -5$$

$$\Rightarrow \alpha = 1, \beta = -12; p = 5, q = -10$$

Hence, the other two vertices are $(1, -12)$, $(5, -10)$.

2. Given equation is $xy - y^2 - x + y = 0$... (i)

Origin is to be shifted at point $(1, 1)$. Putting $x + 1$ in place of x and $y + 1$ in place of y in (i), we get

$$(x+1)(y+1) - (y+1)^2 - (x+1) + (y+1) = 0$$

$$\Rightarrow xy + x + y + 1 - y^2 - 2y - 1 - x - 1 + y + 1 = 0$$

$$\Rightarrow xy - y^2 = 0, \text{ which is the required transformed equation.}$$

3. The given circle is $x^2 + y^2 - 6x + 12y + 15 = 0$

Its centre is $C(3, -6)$ and radius $= \sqrt{3^2 + (-6)^2 - 15}$
 $= \sqrt{30}$.

Since the required circle is concentric with the given circle, so its centre is $C(3, -6)$. Let R be its radius.

$$\text{As given, } \pi R^2 = 2 \times \text{area of given circle} = 2\pi(\sqrt{30})^2$$

$$\Rightarrow R^2 = 60$$

$\therefore$ The equation of the required circle is

$$(x-3)^2 + (y+6)^2 = 60$$

$$\Rightarrow x^2 + y^2 - 6x + 12y - 15 = 0.$$

4. Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } e \text{ be its eccentricity.}$$

$$\text{We have, } e = \frac{5}{8} \text{ and } 2ae = 10.$$

$$\Rightarrow ae = 5 \Rightarrow a = 8$$

$$\text{Now, } b^2 = a^2(1 - e^2) \Rightarrow b^2 = 64\left(1 - \frac{25}{64}\right) = 39$$

$$\text{Hence, length of the latus rectum} = \frac{2b^2}{a} = 2 \times \frac{39}{8} = \frac{39}{4}$$

5. Given points are $P = (a, -8, 4)$ and $Q = (-3, -5, 4)$

$$\therefore PQ^2 = (a+3)^2 + (-8+5)^2 + (4-4)^2 \quad \dots (i)$$

$$\text{Given, } PQ = 5 \Rightarrow PQ^2 = 25$$

$$\therefore \text{From (i), } (a+3)^2 + (-3)^2 + (0)^2 = 25$$

$$\Rightarrow (a+3)^2 = 25 - 9 = 16$$

$$\Rightarrow a+3 = \pm 4 \Rightarrow a = 1, -7$$

6. Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1 \quad \dots (i)$

Its intercepts on x and y axes are a and b respectively.

$$\text{It is given that } \frac{1}{a} + \frac{1}{b} = \text{Constant} = k(\text{say})$$

$$\therefore \frac{1}{ka} + \frac{1}{kb} = 1 \Rightarrow \frac{1}{a} + \frac{1}{b} = 1$$

$$\Rightarrow \left(\frac{1}{k}, \frac{1}{k}\right) \text{ satisfies the equation } \frac{x}{a} + \frac{y}{b} = 1$$

Hence, line (i) passes through the fixed point $\left(\frac{1}{k}, \frac{1}{k}\right)$.

7. The x -coordinates of the points of intersection of the line $lx + my + n = 0$ or $y = -\left(\frac{lx+n}{m}\right)$ and the parabola $y^2 = 4ax$ are roots of the equation.

$$\therefore \left\{-\left(\frac{lx+n}{m}\right)\right\}^2 = 4ax$$

$$\Rightarrow l^2x^2 + 2x(ln - 2am^2) + n^2 = 0$$

If the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$, then this equation has equal roots.

$$\therefore 4(ln - 2am^2)^2 - 4l^2n^2 = 0$$

$$\Rightarrow -4a ln^2m + 4a^2m^4 = 0 \Rightarrow ln = am^2$$

8. Let $A \equiv (0, 4, 1)$, $B \equiv (2, 3, -1)$, $C \equiv (4, 5, 0)$ and $D \equiv (2, 6, 2)$.

Now, $AB^2 = (0-2)^2 + (4-3)^2 + (1+1)^2 = 9 \Rightarrow AB = 3$
Similarly, $CD = 3$, $AD = 3$ and $BC = 3$.

Since, $AB = BC = CD = AD$, hence $ABCD$ is a rhombus.

Also, $AC^2 = (0-4)^2 + (4-5)^2 + (1-0)^2 = 18$

and $BD^2 = (2-2)^2 + (3-6)^2 + (-1-2)^2 = 18$

$\therefore AC = BD$

Hence, $ABCD$ is a square.

9. The given lines are $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$
i.e., $bx + ay - ab = 0$... (i)

and $bx - ay - ab = 0$... (ii)

Let m_1 and m_2 be the slopes of lines (i) and (ii)

respectively, then $m_1 = -\frac{b}{a}$ and $m_2 = \frac{b}{a}$.

If θ is an angle between given lines, then

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2} = \pm \frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)} = \pm \frac{-\frac{2b}{a}}{1 - \frac{b^2}{a^2}}$$

$$= \pm \frac{2ab}{a^2 - b^2}$$

$\therefore$ The tangent of an angle between given lines $= \frac{2ab}{a^2 - b^2}$.

10. Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let e be its eccentricity.

It is given that $\frac{2b^2}{a} = \frac{1}{2}(2b)$

$$\Rightarrow 2b = a \Rightarrow 4b^2 = a^2$$

$$\Rightarrow 4a^2(1 - e^2) = a^2 \Rightarrow 4 - 4e^2 = 1$$

$$\Rightarrow 4e^2 = 3 \Rightarrow e = \frac{\sqrt{3}}{2}$$

Hence, the eccentricity is $\frac{\sqrt{3}}{2}$.

11. Let the given line be AB . Equation of line AB is $2x - y = 5$... (i)

Let P be a point on line AB whose ordinate is equal to abscissa, i.e., $P \equiv (\alpha, \alpha)$

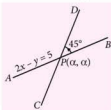
Since $P(\alpha, \alpha)$ lies on line (i)

$$\therefore 2\alpha - \alpha = 5 \Rightarrow \alpha = 5$$

$$\therefore P \equiv (5, 5)$$

Slope of line (i) $= 2 = \tan \theta$ (say)

Let CD be the line passing through $P(5,5)$ and making an



angle of 45° with AB in anticlockwise direction.

Then, slope of $CD = \tan(\theta + 45^\circ)$

$$= \frac{\tan \theta + \tan 45^\circ}{1 - \tan \theta \tan 45^\circ} = \frac{2 + 1}{1 - 2 \cdot 1} = -3$$

$\therefore$ Equation of required line CD is

$$y - 5 = -3(x - 5) \Rightarrow 3x + y - 20 = 0.$$

12. Clearly, the slope of each of the given lines is same equal to $5/2$. Hence, the line $5x - 2y - 1 = 0$ is parallel to each of the given lines.

In order to prove that the line $5x - 2y - 1 = 0$ is mid-parallel to the given lines it is sufficient to show that the line $5x - 2y - 1 = 0$ is equidistant from the given lines.

Putting $y = 0$ in $5x - 2y - 1 = 0$, we get $x = 1/5$. So, the coordinates of a point on $5x - 2y - 1 = 0$ are $(1/5, 0)$.

Let d_1 be the distance between the lines $5x - 2y - 1 = 0$ and $5x - 2y - 9 = 0$.

$\therefore d_1 =$ Length of the perpendicular from $(1/5, 0)$ to $5x - 2y - 9 = 0$

$$\Rightarrow d_1 = \frac{|5 \times (1/5) - 2 \times 0 - 9|}{\sqrt{5^2 + (-2)^2}} = \frac{8}{\sqrt{29}}$$

Let d_2 be the distance between the lines $5x - 2y - 1 = 0$ and $5x - 2y + 7 = 0$.

$\therefore d_2 =$ Length of the perpendicular from $(1/5, 0)$ to $5x - 2y + 7 = 0$

$$\Rightarrow d_2 = \frac{|5 \times (1/5) - 2 \times 0 + 7|}{\sqrt{5^2 + (-2)^2}} = \frac{8}{\sqrt{29}}$$

Clearly, $d_1 = d_2$. Consequently the line $5x - 2y - 1 = 0$ is equidistant from the lines $5x - 2y - 9 = 0$ and $5x - 2y + 7 = 0$.

13. Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$... (i)

Given $e = \sqrt{5}$ and $a + b = 9$

$$\Rightarrow \frac{c}{a} = \sqrt{5} \text{ i.e., } c = \sqrt{5}a \text{ and } b = 9 - a$$

We know that $c^2 = a^2 + b^2$

$$\Rightarrow (\sqrt{5}a)^2 = a^2 + (9 - a)^2 \Rightarrow 5a^2 = a^2 + 81 - 18a + a^2$$

$$\Rightarrow 3a^2 + 18a - 81 = 0 \Rightarrow a^2 + 6a - 27 = 0$$

$$\Rightarrow (a - 3)(a + 9) = 0 \Rightarrow a = 3, -9 \text{ but } a > 0$$

$$\Rightarrow a = 3$$

$$\therefore b = 9 - 3 = 6.$$

Substituting the values of a and b in (i), the equation of the hyperbola is

$$\frac{x^2}{3^2} - \frac{y^2}{6^2} = 1 \text{ i.e., } 4x^2 - y^2 = 36.$$

14. As the ray of light passing through the point $(1, 2)$ reflects on the x -axis at the point A and the reflected ray passes through the point $Q(5, 3)$, $\angle PAN = \angle NAQ$, where AN is perpendicular on x -axis at A .

$\therefore \angle PAB = \angle QAC$.

Let $\angle QAC = \theta$, then

$\angle PAC = \pi - \theta$.

As point A lies on x -axis, let its coordinates be $(a, 0)$.

We know that

$$\tan(\pi - \theta) = -\tan \theta$$

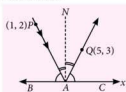
$\Rightarrow$ slope of $AP = -\text{slope of } AQ$

$$\Rightarrow \frac{2-0}{1-a} = -\frac{3-0}{5-a} \Rightarrow \frac{2}{1-a} = -\frac{3}{5-a}$$

$$\Rightarrow -10 + 2a = 3 - 3a$$

$$\Rightarrow 5a = 13 \Rightarrow a = \frac{13}{5}$$

Hence, the coordinates of the point A are $\left(\frac{13}{5}, 0\right)$



15. Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

...(i)

Since circle (i) touches y -axis at $(0, 3)$

$$\therefore 9 + 6f + c = 0 \text{ and } f^2 = c$$

$$\therefore f^2 + 6f + 9 = 0 \Rightarrow (f+3)^2 = 0 \Rightarrow f = -3$$

$$\therefore c = f^2 = 9$$

Since, length of the intercept of circle (i) on x -axis is 8

$$\therefore 2\sqrt{g^2 - c} = 8 \Rightarrow g^2 - c = 16$$

$$\Rightarrow g^2 = c + 16 = f^2 + 16 = 9 + 16 = 25 \therefore g = \pm 5$$

Thus centre of the required circles are $(-g, -f)$ i.e., $(-5, 3)$

and $(5, 3)$ and radius = $\sqrt{g^2 + f^2 - c} = \sqrt{25 + 9 - 9} = 5$

Hence, equation of required circles are

$$(x+5)^2 + (y-3)^2 = 5^2 \text{ and } (x-5)^2 + (y-3)^2 = 5^2$$

$$\text{i.e., } x^2 + y^2 + 10x - 6y + 9 = 0 \text{ and } x^2 + y^2 - 10x - 6y + 9 = 0$$

16. The equation of the variable line AB is $x \cos \alpha + y \sin \alpha = p$, where α is variable and p is constant.

Reducing it to intercept form, we get

$$\frac{x}{\frac{p}{\cos \alpha}} + \frac{y}{\frac{p}{\sin \alpha}} = 1$$

So, the coordinates of

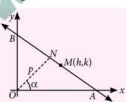
the points A and B are

$$\left(\frac{p}{\cos \alpha}, 0\right) \text{ and } \left(0, \frac{p}{\sin \alpha}\right)$$

respectively.

Let $M(h, k)$ be the mid-point of the segment AB , then

$$h = \frac{\frac{p}{\cos \alpha} + 0}{2} \text{ and } k = \frac{0 + \frac{p}{\sin \alpha}}{2}$$



$$\Rightarrow 2h = \frac{p}{\cos \alpha} \text{ and } 2k = \frac{p}{\sin \alpha}$$

$$\Rightarrow \cos \alpha = \frac{p}{2h} \text{ and } \sin \alpha = \frac{p}{2k}$$

To eliminate α , squaring and adding these, we get

$$\cos^2 \alpha + \sin^2 \alpha = \frac{p^2}{4h^2} + \frac{p^2}{4k^2}$$

$$\Rightarrow 1 = \frac{p^2}{4h^2} + \frac{p^2}{4k^2} \Rightarrow \frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$$

$\therefore$ The locus of the point $M(h, k)$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

17. Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be the vertices of ΔABC such that $D(5, 7, 11)$, $E(0, 8, 5)$ and $F(2, 3, -1)$ be the mid-points of the sides BC , CA and AB respectively.

We know that $AFDE$ is a parallelogram.

Mid-point of AD is $\left(\frac{x_1+5}{2}, \frac{y_1+7}{2}, \frac{z_1+11}{2}\right)$ and mid-

point of EF is $\left(\frac{2+0}{2}, \frac{3+8}{2}, \frac{-1+5}{2}\right)$.

Since the diagonals of a parallelogram bisect each other, so mid-points of AD and EF are same.

$$\therefore \frac{x_1+5}{2} = \frac{2+0}{2}, \frac{y_1+7}{2} = \frac{3+8}{2}, \frac{z_1+11}{2} = \frac{-1+5}{2}$$

$$\Rightarrow x_1 + 5 = 2, y_1 + 7 = 11, z_1 + 11 = 4$$

$$\Rightarrow x_1 = -3, y_1 = 4, z_1 = -7.$$

$\therefore$ The vertex A is $(-3, 4, -7)$.

Also, $BDEF$ is a parallelogram. So mid-points of BE and FD are same.

$$\therefore \frac{x_2+0}{2} = \frac{2+5}{2}, \frac{y_2+8}{2} = \frac{3+7}{2}, \frac{z_2+5}{2} = \frac{-1+11}{2}$$

$$\Rightarrow x_2 = 7, y_2 + 8 = 10, z_2 + 5 = 10 \Rightarrow x_2 = 7, y_2 = 2, z_2 = 5.$$

$\therefore$ The vertex B is $(7, 2, 5)$.

Again, $DCEF$ is a parallelogram. So mid-points of CF and DE are same.

$$\therefore \frac{x_3+2}{2} = \frac{5+0}{2}, \frac{y_3+3}{2} = \frac{7+8}{2}, \frac{z_3+(-1)}{2} = \frac{11+5}{2}$$

$$\Rightarrow x_3 + 2 = 5, y_3 + 3 = 15, z_3 - 1 = 16 \Rightarrow x_3 = 3, y_3 = 12, z_3 = 17.$$

$\therefore$ The vertex C is $(3, 12, 17)$.

18. Since man just touches the top of the arch,

$\therefore$ Greatest height of the arch = $OR = b$

Let equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots(i)$$

Given, $A'A = 30$

$$\Rightarrow A'O = 15 = OA = a$$

Let $AM = 2$

Then, $OM = 13$ and

$MP = 6$ (Given)

Now, $P = (13, 6)$

Since, $P(13, 6)$ lies on (i)

$$\therefore \frac{(13)^2}{(15)^2} + \frac{(6)^2}{b^2} = 1$$

$$\Rightarrow \frac{36}{b^2} = 1 - \frac{169}{225} = \frac{225 - 169}{225} = \frac{56}{225}$$

$$\Rightarrow b^2 = \frac{(36)(225)}{56} = \frac{(9)(225)}{14}$$

$$\Rightarrow b = \frac{(3)(15)}{\sqrt{14}} = \frac{45}{\sqrt{14}}$$

$$\therefore b = \frac{45\sqrt{14}}{14} \text{ ft.}$$

19. Let P and Q be the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ respectively. Then the equation of the line PQ is

$$y - 2at_1 = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} (x - at_1^2)$$

$$\Rightarrow y - 2at_1 = \frac{2}{t_1 + t_2} (x - at_1^2)$$

$$\Rightarrow y(t_1 + t_2) - 2at_1^2 - 2at_1t_2 = 2x - 2at_1^2$$

$$\Rightarrow y(t_1 + t_2) = 2(x + at_1t_2) \quad \dots(i)$$

Since PQ is a focal chord, it passes through focus $F(a, 0)$ i.e., $F(a, 0)$ lies on (i).

$$\therefore 0 \cdot (t_1 + t_2) = 2(a + at_1t_2)$$

$$\Rightarrow a(1 + t_1t_2) = 0 \Rightarrow t_1t_2 = -1 \quad \dots(ii)$$

$$\text{Now, } |FP| = \sqrt{(at_1^2 - a)^2 + (2at_1 - 0)^2} = a\sqrt{(t_1^2 - 1)^2 + 4t_1^2}$$

$$= a\sqrt{(t_1^2 + 1)^2} = a(1 + t_1^2)$$

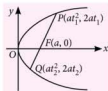
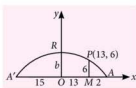
Similarly, $|FQ| = a(1 + t_2^2)$

$$\therefore \frac{1}{|FP|} + \frac{1}{|FQ|} = \frac{1}{a(1 + t_1^2)} + \frac{1}{a(1 + t_2^2)} = \frac{1 + t_2^2 + 1 + t_1^2}{a(1 + t_1^2)(1 + t_2^2)}$$

$$= \frac{2 + t_1^2 + t_2^2}{a(1 + t_1^2 + t_2^2 + t_1^2t_2^2)} = \frac{2 + t_1^2 + t_2^2}{a(1 + t_1^2 + t_2^2 + 1)} \quad (\text{using (ii)})$$

$$= \frac{2 + t_1^2 + t_2^2}{a(2 + t_1^2 + t_2^2)} = \frac{1}{a} = \frac{2}{2a}$$

$$= \frac{2}{l} \quad \left(\because l = \frac{1}{2} (\text{length of latus-rectum}) = \frac{1}{2} \cdot 4a = 2a \right)$$



Hence, $\frac{1}{|FP|} + \frac{1}{|FQ|} = \frac{2}{l}$, where l is length of semi-latus rectum.

20. Let the equation of lines AB and CD be

$$5x - y + 4 = 0 \quad \dots(i)$$

$$\text{and } 3x + 4y - 4 = 0 \quad \dots(ii)$$

To take any point on line (i), let $x = \alpha$, then from (i),

$$y = 5\alpha + 4$$

Let $Q = (\alpha, 5\alpha + 4)$ and

$$R = \left(\beta, \frac{4 - 3\beta}{4} \right)$$

Let $P \equiv (1, 5)$

Since, $P(1, 5)$ is the middle point of QR

$$\therefore \frac{\alpha + \beta}{2} = 1 \Rightarrow \alpha + \beta = 2 \quad \dots(iii)$$

$$\text{and } \frac{5\alpha + 4 + \frac{4 - 3\beta}{4}}{2} = 5 \Rightarrow 20\alpha - 3\beta = 20 \quad \dots(iv)$$

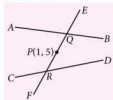
Solving (iii) and (iv), we get $\alpha = \frac{26}{23}$ and $\beta = \frac{20}{23}$

$$\therefore Q = \left(\frac{26}{23}, \frac{222}{23} \right)$$

$\therefore$ Equation of line EF is

$$y - 5 = \frac{5 - \frac{222}{23}}{1 - \frac{26}{23}} (x - 1) \Rightarrow y - 5 = \frac{107}{3} (x - 1)$$

$$\Rightarrow 107x - 3y - 92 = 0.$$



PUZZLE CORNER

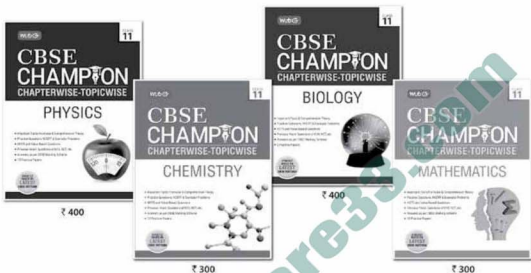
SOLUTION - OCTOBER 2018



12+	1	6	8+	4	2	5	5	10+	3
	5	3	3	2	4	4	6	1	
12+	4	5	3	9+	1	2	6		
10+	6	2	12+	5	3	3	1	9+	4
	2	4	8+	4	1	6	7+	3	5
3	1	6	11+	6	5	4	2		

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MPP-7

MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.



Limits and Derivatives

Total Marks : 80

Time Taken : 60 Min.

Only One Option Correct Type

1. $\lim_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{e^{1/n}}{n} + \frac{e^{2/n}}{n} + \dots + \frac{e^{(n-1)/n}}{n} \right)$ equals

(a) 0 (b) 1
(c) $e - 1$ (d) None of these

2. The value of $\lim_{x \rightarrow 0} \left\{ \left[\frac{100x}{\sin x} \right] + \left[\frac{99 \sin x}{x} \right] \right\}$, where $\{ \}$ represents the greatest integer function, is

(a) 199 (b) 198
(c) 0 (d) None of these

3. $\lim_{n \rightarrow \infty} \left\{ \frac{n^2 - n + 1}{n^2 - n - 1} \right\}^{n(n-1)}$ is equal to

(a) e (b) e^2
(c) e^{-1} (d) 1

4. The value of $\lim_{x \rightarrow 0} \frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}$, $a > 1$, is

(a) 4 (b) 2
(c) -1 (d) 0

5. If $y = \sqrt{\log x} + \sqrt{\log x} + \sqrt{\log x} + \dots$, then $\frac{dy}{dx}$ is

(a) $\frac{x}{2y-1}$ (b) $\frac{x}{2y+1}$
(c) $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

6. $\lim_{x \rightarrow \infty} \left\{ \frac{1^2}{1-x^3} + \frac{3}{1+x^2} + \frac{5^2}{1-x^3} + \frac{7}{1+x^2} + \dots \right\}$ equal to

(a) $-\frac{5}{6}$ (b) $-\frac{10}{3}$
(c) $\frac{5}{6}$ (d) None of these

One or More Than One Option(s) Correct Type

7. If $\lim_{n \rightarrow \infty} \left(an - \frac{1+n^2}{1+n} \right) = b$, where a is a finite number, then

(a) $a = 1$ (b) $a = 0$
(c) $b = 1$ (d) $b = -1$

8. $L = \lim_{x \rightarrow a} \frac{2 \sin x - 1}{2 \sin x - 1}$, then

(a) limit does not exist when $a = \pi/6$
(b) $L = -1$ when $a = \pi$
(c) $L = 1$ when $a = \pi/2$
(d) $L = 1$ when $a = 0$

9. If $\lim_{x \rightarrow 0} \frac{a \sin x - bx + cx^2 + x^3}{2x^2 \log(1+x) - 2x^3 + x^4}$ exists and is finite, then

(a) $a = b$ (b) $c = 0$
(c) $a = 6$ (d) $c = 2$

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10. If $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$, then

- (a) $b - a = 5/6$ (b) $a = -5/2$
(c) $a - b = -1$ (d) $b = 3/2$

11. If $f(\alpha) = \lim_{x \rightarrow 2} (\sin^x \alpha + \cos^x \alpha)^{\frac{1}{(x-2)}}$ for $\alpha \in \left[0, \frac{\pi}{2}\right]$, then

- (a) $f(0) = 1$
(b) $f\left(\frac{\pi}{2}\right) = 1$
(c) $f(\alpha) = (\cos \alpha)^{\cos^2 \alpha} \cdot (\sin \alpha)^{\sin^2 \alpha}$ if $\alpha \in \left(0, \frac{\pi}{2}\right)$
(d) $f(\alpha) = \frac{(\sin \alpha)^{\sin^2 \alpha}}{(\cos \alpha)^{\cos^2 \alpha}}$ if $\alpha \in \left(0, \frac{\pi}{2}\right)$

12. If $L = \lim_{x \rightarrow 0} \frac{1}{x^3} \left(\frac{1}{\sqrt{1+x}} - \frac{1+ax}{1+bx} \right)$ exists, then

- (a) $a = \frac{1}{4}$ (b) $b = \frac{3}{4}$
(c) $L = -\frac{1}{32}$ (d) $L = \frac{1}{32}$

13. If $A = \lim_{x \rightarrow -2} \frac{\tan \pi x}{x+2} + \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)^x$, then

- (a) $A > 3$ (b) $A > 4$
(c) $A < 4$
(d) A is a transcendental number

Comprehension Type

If $f(x) = \lim_{n \rightarrow \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}}$ and $\lim_{x \rightarrow \pm 1} f(x)$ exists, then

14. The value of a is

- (a) -1 (b) 1
(c) 0 (d) 2

15. The value of b is

- (a) -1 (b) 1
(c) 0 (d) 2

Matrix Match Type

16. Match the following columns:

Column-I		Column-II	
P.	If $L = \lim_{x \rightarrow 1} \frac{\sqrt[3]{7-x} - 2}{(x+1)}$, then $12L =$	1.	-2
Q.	If $L = \lim_{x \rightarrow \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$, then $-L/4 =$	2.	2
R.	If $L = \lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2 + x - 3}$, then $20L =$	3.	1
S.	If $L = \lim_{x \rightarrow \infty} \frac{\log x^4 - [x]}{[x]}$, where $n \in N$, ($[x]$ denotes greatest integer less than or equal to x), then $-2L =$	4.	-1

P	Q	R	S
(a) 1	2	4	3
(b) 4	1	3	2
(c) 4	3	1	2
(d) 3	1	2	4

Integer Answer Type

17. The value of $\lim_{x \rightarrow \pi/4} \frac{8 - \sqrt{2}(\cos x + \sin x)^5}{1 - \sin 2x}$ is

18. If $y = \frac{a + bx^{3/2}}{x^{5/4}}$ and $y' = 0$ at $x = 5$, then the value of a^2/b^2 is

19. Let $y = \frac{2^{\log_{3/4} x} - 3^{\log_{27}(x^2+1)^3} - 2x}{7^{\log_{49} x} - x - 1}$ and $\frac{dy}{dx} = ax + b$, then the value of $a + b$ is

20. If $xe^{xy} = y + \sin^2 x$, then at $x = 0$, $dy/dx =$

••

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted
No. of questions correct
Marks scored in percentage

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY !	Revise thoroughly and strengthen your concepts.

CLASS XII Series 7

CBSE DRILL



Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Vector Algebra

VECTOR

- A quantity which has magnitude as well as direction is called a vector quantity or simply a vector.

REPRESENTATION OF VECTOR

- A vector is determined by two points A and B such that the magnitude of the vector is the length of the segment AB and its direction is from A to B . Thus, a vector is written as $\overrightarrow{AB}$, where A is called its origin or the initial point and B is called its terminal point or end point.

MAGNITUDE OF A VECTOR

- The magnitude of a vector $\overrightarrow{AB}$ is the length of the line segment representing it. So, magnitude will be non-negative quantity. It is represented by $|\overrightarrow{AB}|$.

POSITION VECTOR

- If P is a point in space having coordinates (x, y, z) w.r.t. the origin $O(0, 0, 0)$, then the vector $\overrightarrow{OP}$ is called the position vector of the point P . The magnitude of $\overrightarrow{OP}$ is, $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

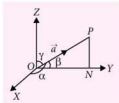
DIRECTION COSINES AND DIRECTION RATIOS OF A VECTOR

- If a vector $\vec{a}$ makes angles α, β, γ with the positive direction of X, Y and Z axes respectively, then $\cos \alpha, \cos \beta,$

$\cos \gamma$ are called direction cosines of vector $\vec{a}$ and are usually denoted by l, m, n .

- Any three numbers a, b, c proportional to the direction cosines l, m, n respectively of a line are called direction ratios of the line.

$$\therefore \frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$



TYPES OF VECTORS

- Zero or null vector :** A vector whose magnitude is 0 and direction is indeterminate.

Representation : $\vec{0}$

- Unit Vector :** A vector whose magnitude is unity.

Representation : $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

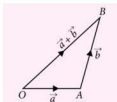
- Like and Unlike Vectors :** Vectors are said to be like when they have same direction and unlike when they have opposite direction.

- Collinear or Parallel Vectors :** Vectors parallel to the same line irrespective of their magnitudes and directions.

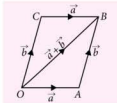
- **Coinitial Vectors** : Vectors with same initial point.
- **Equal Vectors** : Vectors having equal magnitudes and same direction.
- **Negative of a Vector** : Vectors having same magnitude but opposite in direction.
- **Coplanar Vectors** : Vectors lie in the same plane or parallel to the same plane.

ADDITION OF VECTORS

- **Triangle Law** : If two vectors $\vec{a}$ and $\vec{b}$ lie along the two sides of a triangle in consecutive order, then third side represents the sum $\vec{a} + \vec{b}$.



- **Parallelogram Law** : If two vectors lie along two adjacent sides of a parallelogram, then diagonal of the parallelogram through the common vertex represents their sum i.e., $\vec{OB} = \vec{a} + \vec{b}$.



Properties of Vector Addition

- **Commutative Law** : $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
- **Associative Law** : $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
- **Existence of Additive Identity** : $\vec{a} + \vec{0} = \vec{a} = \vec{0} + \vec{a}$ i.e., $\vec{0}$ is the additive identity.
- **Existence of Additive Inverse** : $\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$ i.e., $(-\vec{a})$ is the additive inverse of $\vec{a}$.

MULTIPLICATION OF A VECTOR BY A SCALAR

- If $\vec{a}$ is a vector and m is a scalar, then $m\vec{a}$ is a vector parallel to $\vec{a}$ whose magnitude is $|m|$ times to that of $\vec{a}$. This multiplication is called scalar multiplication.

Properties of Multiplication of a Vector by a Scalar

- **Commutative Law** : $m(\vec{a}) = (\vec{a})m = m\vec{a}$
- **Associative Law** : $m(n\vec{a}) = n(m\vec{a}) = mn(\vec{a})$
- **Distributive Law** : $(m+n)\vec{a} = m\vec{a} + n\vec{a}$ and $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$

COMPONENTS OF A VECTOR

- If $P(x, y, z)$ is a point in space and $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along X, Y and Z-axes respectively, then the

vector components of $\vec{OP}$ along X, Y and Z-axes respectively are $x\hat{i}$, $y\hat{j}$ and $z\hat{k}$. Also, x , y and z are called scalar components of $\vec{OP}$.

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow |\vec{OP}| = \sqrt{x^2 + y^2 + z^2}$$

VECTORS JOINING TWO POINTS

- If $P \equiv (x_1, y_1, z_1)$ and $Q \equiv (x_2, y_2, z_2)$, are any two points

$$\text{then } \vec{PQ} \equiv (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\text{Also, } |\vec{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

SECTION FORMULA

Let A and B be any two points with position vectors $\vec{a}$ and $\vec{b}$ respectively. Let C($\vec{c}$) divides AB in the ratio $m : n$, then

- **Internal division** : $\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$
- **External division** : $\vec{c} = \frac{m\vec{b} - n\vec{a}}{m-n}$

MID POINT FORMULA

- Let C ($\vec{c}$) is the mid point of AB, then $\vec{c} = \frac{\vec{a} + \vec{b}}{2}$.

SCALAR OR DOT PRODUCT

Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors inclined at an angle θ , then scalar product of $\vec{a}$ and $\vec{b}$ is denoted and defined by $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$.

Properties of Scalar Product

- $\vec{a} \cdot \vec{b}$ is a real number.
- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (Commutative law)
- $(x\vec{a}) \cdot \vec{b} = x(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (x\vec{b})$, where x is a scalar (Associative law)
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ (Distributive law)
- $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$
- If $\vec{a}$ is perpendicular to $\vec{b}$, then $\vec{a} \cdot \vec{b} = 0$
- $\vec{a} \cdot \vec{b} < 0$, if and only if the angle between $\vec{a}$ and $\vec{b}$ is obtuse.
- $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} \perp \vec{b}$.
- $\hat{i} \cdot \hat{i} = 1, \hat{j} \cdot \hat{j} = 1, \hat{k} \cdot \hat{k} = 1, \hat{i} \cdot \hat{j} = 0, \hat{j} \cdot \hat{k} = 0, \hat{i} \cdot \hat{k} = 0$
- If two vectors have same direction, then $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$

- (xi) If two vectors have opposite direction, then $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$
- (xii) If $\hat{a}$ and $\hat{b}$ are unit vectors, then $|\hat{a}| = 1, |\hat{b}| = 1 \therefore \hat{a} \cdot \hat{b} = 1 \cdot 1 \cdot \cos \theta$
- (xiii) If $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$
Then, $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$
- (xiv) If $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$, $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$ and θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, then
- $$\theta = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \right) = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \right)$$
- $$\therefore \theta = \cos^{-1} \left(\frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}} \right)$$

PROJECTION OF A VECTOR ON A LINE

- Projection of a given vector $\vec{a}$ along a directed line l is given by $\frac{\vec{a} \cdot \vec{p}}{|\vec{p}|}$, where $\vec{p}$ is a vector along the direction of the line.
- Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is given by $\vec{a} \cdot \hat{b}$ or $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ or $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

VECTOR OR CROSS PRODUCT

- The vector product of two vectors $\vec{a}$ and $\vec{b}$ is denoted and defined as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin \theta \hat{n}$, where θ be the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a perpendicular unit vector to both $\vec{a}$ and $\vec{b}$.

Properties of Vector Product

- (i) $\vec{a} \times \vec{b}$ is a vector.
- (ii) The angle between two vectors $\vec{a}$ and $\vec{b}$ is given as $\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
- (iii) If $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} \parallel \vec{b}$ or angle between $\vec{a}$ and $\vec{b}$ i.e., $\theta = 0$ or π .
- (iv) If $\theta = \frac{\pi}{2}$, then $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$.
- (v) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$
 $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$
 $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}$ and $\hat{i} \times \hat{k} = -\hat{j}$
- (vi) The vector product is not commutative, as $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$.

- (vii) If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors and λ be a scalar, then

$$(a) \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

[Distributivity of vector product over addition]

$$(b) \lambda(\vec{a} \times \vec{b}) = (\lambda \vec{a}) \times \vec{b} = \vec{a} \times (\lambda \vec{b})$$

- (viii) If $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$,

$$\text{then } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- (ix) If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then its area is given by $\frac{1}{2} |\vec{a} \times \vec{b}|$.

- (x) If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then its area is given by $|\vec{a} \times \vec{b}|$.

- (xi) If $\vec{a}$ and $\vec{b}$ are any two vectors, then

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

SCALAR TRIPLE PRODUCT

- Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors. Then the scalar product of $\vec{a} \times \vec{b}$ and $\vec{c}$ is called scalar triple product of $\vec{a}, \vec{b}$ and $\vec{c}$. It is denoted by $(\vec{a} \times \vec{b}) \cdot \vec{c}$ or $[\vec{a} \vec{b} \vec{c}]$ or $[\vec{a}, \vec{b}, \vec{c}]$. ($\times$ and $\cdot$ may interchange)

Properties of scalar triple product

$$[\vec{a} \vec{b} \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\text{where } \vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}, \vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

$$\text{and } \vec{c} = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}.$$

- $[\vec{a} \vec{b} \vec{c}] = [\vec{b} \vec{c} \vec{a}] = [\vec{c} \vec{a} \vec{b}]$
- $[\vec{a} \vec{b} \vec{c}] = -[\vec{a} \vec{c} \vec{b}]$
- $[\vec{a} \vec{a} \vec{b}] = 0$
- Volume of parallelepiped whose coterminous edges are $\vec{a}, \vec{b}, \vec{c}$ is $|\vec{a} \vec{b} \vec{c}|$.

COPLANARITY OF THREE VECTORS

Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are said to be coplanar iff $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ i.e., $[\vec{a} \vec{b} \vec{c}] = 0$

WORK IT OUT

VERY SHORT ANSWER TYPE

- If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then find $|\vec{a} \times \vec{b}|$.
- Find the value of p for which the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel.

3. If $\overrightarrow{OP} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = 3\hat{i} - 4\hat{j} + 2\hat{k}$, find the modulus and direction cosines of $\overrightarrow{PQ}$.
4. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angles between the vectors $\vec{a}$ and $\vec{b}$.
5. If $\vec{a} = a_1\hat{i} + a_2\hat{j}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j}$ are two non-zero vectors, then prove that they are parallel if and only if $a_1 b_2 - a_2 b_1 = 0$.

SHORT ANSWER TYPE

6. A vector $\overrightarrow{OP}$ is inclined to OX at 45° and OY at 60° . Find the angle at which $\overrightarrow{OP}$ is inclined to OZ .
7. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of points A, B, C, D respectively and $\vec{b} - \vec{a} = 2(\vec{d} - \vec{c})$, show that the point of intersection of the straight lines AD and BC divide these lines in the ratio $2:1$.
8. Find the values of x for which the vector $\vec{r} = (x^2 - 1)\hat{i} + 5\hat{j} - 3(x^2 - 4)\hat{k}$ makes acute angles with the coordinate axes.
9. If the vectors $\vec{a} = a\hat{i} + a\hat{j} + c\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + b\hat{k}$ are coplanar, then prove that c is the geometric mean of a and b .
10. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\cos \frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$.

LONG ANSWER TYPE - I

11. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$, $\vec{c} = \lambda(\vec{a} \times \vec{b})$ and $|\vec{a}| = \frac{1}{\sqrt{2}}$, $|\vec{b}| = \frac{1}{\sqrt{3}}$, $|\vec{c}| = \frac{1}{\sqrt{6}}$, find the angle between $\vec{a}$ and $\vec{b}$.
12. Prove that $\vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = [\vec{a} \vec{b} \vec{c}]$.
13. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq \vec{0}$, then prove that $\vec{b} = \vec{c}$.
14. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, prove that $(\vec{a} \times \vec{b}) = (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{a})$.
15. Show that a unit vector perpendicular to each of the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$ is $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ and the sine of the angle between them is $\frac{2}{\sqrt{7}}$.

LONG ANSWER TYPE - II

16. Show that the vector of magnitude $\sqrt{51}$ which makes equal angles with vectors $\vec{a} = \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$, $\vec{b} = \frac{1}{5}(-4\hat{i} - 3\hat{k})$ and $\vec{c} = \hat{j}$ is $-\hat{5}\hat{i} + \hat{j} + 5\hat{k}$.
17. For any two vectors $\vec{a}$ and $\vec{b}$, show that $(1 + |\vec{a}|^2)(1 + |\vec{b}|^2) = (1 - \vec{a} \cdot \vec{b})^2 + |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|^2$.
18. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, prove that $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$.
19. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$ and $\vec{b}, \vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.
20. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and the vectors $\vec{A} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{B} = \hat{i} + b\hat{j} + b^2\hat{k}$, $\vec{C} = \hat{i} + c\hat{j} + c^2\hat{k}$ are non-coplanar, then prove that $abc = -1$.

SOLUTIONS

1. We know that $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$
 $= 10^2 \cdot 2^2 - 12^2 = 400 - 144 = 256$
 $\left[\text{Given } |\vec{a}| = 10, |\vec{b}| = 2, \vec{a} \cdot \vec{b} = 12 \right]$
 $\Rightarrow |\vec{a} \times \vec{b}|^2 = 256 \Rightarrow |\vec{a} \times \vec{b}| = 16$
2. We know that vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are parallel iff $\vec{a} = \lambda\vec{b}$
 Here, $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$ are parallel
 $\therefore \frac{3}{1} = \frac{2}{p} = \frac{9}{3} \Rightarrow \frac{2}{p} = 3 \Rightarrow p = \frac{2}{3}$
3. Given, $\overrightarrow{OP} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = 3\hat{i} - 4\hat{j} + 2\hat{k}$
 Now, $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (3\hat{i} - 4\hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} - \hat{k})$
 $= \hat{i} - 7\hat{j} + 3\hat{k}$
 $\therefore |\overrightarrow{PQ}| = \sqrt{1^2 + (-7)^2 + 3^2} = \sqrt{59}$
 Now unit vector along $\overrightarrow{PQ}$
 $= \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{\hat{i} - 7\hat{j} + 3\hat{k}}{\sqrt{59}} = \frac{1}{\sqrt{59}}\hat{i} - \frac{7}{\sqrt{59}}\hat{j} + \frac{3}{\sqrt{59}}\hat{k}$

Hence, direction cosines of $\overrightarrow{PQ}$ are $\frac{1}{\sqrt{59}}, -\frac{7}{\sqrt{59}}, \frac{3}{\sqrt{59}}$.

4. Let θ ($0 \leq \theta \leq \pi$) be the angle between the vectors $\vec{a}$ and $\vec{b}$.

$$\begin{aligned} \text{Given } \vec{a} \times \vec{b} \text{ is a unit vector} &\Rightarrow |\vec{a} \times \vec{b}| = 1 \Rightarrow |\vec{a}| \cdot |\vec{b}| \sin \theta = 1 \\ \Rightarrow 3 \cdot \frac{\sqrt{2}}{3} \sin \theta &= 1 \quad \left[\because |\vec{a}| = 3 \text{ and } |\vec{b}| = \frac{\sqrt{2}}{3} \right] \\ \Rightarrow \sin \theta &= \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4} \quad (\because 0 \leq \theta \leq \pi) \end{aligned}$$

5. We know that the vectors $\vec{a}$ and $\vec{b}$ are parallel iff there exists a (non-zero) scalar λ such that $\vec{a} = \lambda \vec{b}$

$$\begin{aligned} \Leftrightarrow a_1 \hat{i} + a_2 \hat{j} &= \lambda (b_1 \hat{i} + b_2 \hat{j}) \\ \Leftrightarrow a_1 \hat{i} + a_2 \hat{j} &= \lambda b_1 \hat{i} + \lambda b_2 \hat{j} \\ \Leftrightarrow a_1 &= \lambda b_1 \text{ and } a_2 = \lambda b_2 \\ \Leftrightarrow \frac{a_1}{a_2} &= \frac{b_1}{b_2} \Leftrightarrow a_1 b_2 - a_2 b_1 = 0. \end{aligned}$$

Hence the given vectors are parallel if and only if $a_1 b_2 - a_2 b_1 = 0$.

6. Suppose $\overrightarrow{OP}$ is inclined at angle γ to OZ . Let l, m, n be the direction cosines of $\overrightarrow{OP}$.

Then, $l = \cos 45^\circ, m = \cos 60^\circ, n = \cos \gamma$

$$\Rightarrow l = \frac{1}{\sqrt{2}}, m = \frac{1}{2}, n = \cos \gamma.$$

$$\text{Now, } l^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + n^2 = 1$$

$$\Rightarrow n^2 = \frac{1}{4} \Rightarrow n = \pm \frac{1}{2} \Rightarrow \cos \gamma = \pm \frac{1}{2} \Rightarrow \gamma = 60^\circ \text{ or } 120^\circ$$

Hence, $\overrightarrow{OP}$ is inclined at angle of 60° or 120° to OZ .

7. Let P and Q be points which divide AD and BC respectively in the ratio $2 : 1$.

Let the position vector of P and Q be $\vec{a}$ and $\vec{b}$, respectively, then

$$\vec{a} = \frac{2\vec{d} + \vec{a}}{2+1} = \frac{2\vec{d} + \vec{a}}{3} \quad \dots(i)$$

$$\text{and } \vec{b} = \frac{2\vec{c} + \vec{b}}{2+1} = \frac{2\vec{c} + \vec{b}}{3} \quad \dots(ii)$$

$$\text{Given, } \vec{b} - \vec{a} = 2(\vec{d} - \vec{c}) \Rightarrow \vec{b} = \vec{a} + 2\vec{d} - 2\vec{c} \quad \dots(iii)$$

Putting this value of $\vec{b}$ in (ii), we get

$$\vec{b} = \frac{2\vec{c} + \vec{a} + 2\vec{d} - 2\vec{c}}{3} = \frac{2\vec{d} + \vec{a}}{3} = \vec{a} \quad [\text{From (i)}]$$

Hence P and Q are the same points and point of intersection of AD and BC divides these lines in the ratio $2 : 1$.

8. Since $\vec{r}$ makes acute angles with the coordinate axes, i.e., $\vec{r} \cdot \vec{i} > 0, \vec{r} \cdot \vec{j} > 0$ and $\vec{r} \cdot \vec{k} > 0$.

$$\Rightarrow (x^2 - 1) \cdot 1 > 0, 5 \cdot 1 > 0 \text{ and } 3(x^2 - 4) > 0$$

$$\Rightarrow x^2 - 1 > 0, 5 > 0 \text{ and } x^2 - 4 < 0$$

$$\Rightarrow (x+1)(x-1) > 0 \text{ and } (x+2)(x-2) < 0$$

$$\Rightarrow (x < -1 \text{ or } x > 1) \text{ and } -2 < x < 2$$

$$\Rightarrow x \in (-2, -1) \cup (1, 2).$$

9. Since $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are coplanar vectors, then

$$[\vec{\alpha} \vec{\beta} \vec{\gamma}] = 0$$

$$\Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\Rightarrow a(0 - c) - a(b - c) + c(c - 0) = 0$$

$$\Rightarrow -ac - ab + ac + c^2 = 0$$

$$\Rightarrow c^2 = ab \Rightarrow c \text{ is the geometric mean of } a \text{ and } b.$$

10. Here, $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = 1 \cdot 1 \cos \theta = \cos \theta$

$$[\because |\vec{a}| = 1 = |\vec{b}|]$$

$$\text{Now, } (\vec{a} + \vec{b})^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1 + 1 + 2 \cos \theta$$

$$= 2(1 + \cos \theta) = 2 \left(2 \cos^2 \frac{\theta}{2} \right)$$

$$\text{Also, } |\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2 = 4 \cos^2 \frac{\theta}{2}$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{1}{2} |\vec{a} + \vec{b}|.$$



COMIC CAPSULE

Be Rational!

$\sqrt{-1}$

Get Real!

π

11. We have, $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$\Rightarrow \vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$$\Rightarrow \vec{c} \cdot \vec{a} = \vec{c} \cdot \vec{b} = 0$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$$\text{As given, } |\vec{a} + \vec{b} + \vec{c}| = 1$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 1$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + 2\left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} \cos \theta + 0 + 0\right) = 1 \quad [\text{From (i)}]$$

$$\Rightarrow 1 + 2\left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} \cos \theta\right) = 1$$

$$\Rightarrow \frac{\sqrt{2}}{3} \cos \theta = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Hence, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

12. Consider $\vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c})$

$$= \vec{a} \cdot (\vec{b} \times \vec{a} + \vec{b} \times 2\vec{b} + \vec{b} \times 3\vec{c} + \vec{c} \times \vec{a} + \vec{c} \times 2\vec{b} + \vec{c} \times 3\vec{c})$$

$$= \vec{a} \cdot (\vec{b} \times \vec{a} + 3\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + 2\vec{c} \times \vec{b})$$

$$[\because \vec{b} \times 2\vec{b} = \vec{0}, \vec{c} \times 3\vec{c} = \vec{0}]$$

$$= \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (3\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{a} \cdot (2\vec{c} \times \vec{b})$$

$$= 0 + 3(\vec{a} \cdot (\vec{b} \times \vec{c})) + 0 + 2(\vec{a} \cdot (\vec{c} \times \vec{b}))$$

$$= 3[\vec{a} \vec{b} \vec{c}] + 2[\vec{a} \vec{c} \vec{b}] = 3[\vec{a} \vec{b} \vec{c}] - 2[\vec{a} \vec{b} \vec{c}] = [\vec{a} \vec{b} \vec{c}]$$

13. Given $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$$\Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} - \vec{c} = \vec{0} \text{ or } \vec{a} \perp (\vec{b} - \vec{c})$$

But $\vec{a} \neq \vec{0}$

(Given)

$$\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \perp (\vec{b} - \vec{c}) \quad \dots(i)$$

$$\text{Also, } \vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$$

$$\Rightarrow \vec{a} = \vec{0} \text{ or } \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c})$$

$$\text{But } \vec{a} \neq \vec{0} \Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c}) \quad \dots(ii)$$

From (i) and (ii), we get $(\vec{b} = \vec{c})$

$(\because \vec{a} \perp (\vec{b} - \vec{c}) \text{ and } \vec{a} \parallel (\vec{b} - \vec{c}) \text{ cannot hold simultaneously})$

14. Given, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} + \vec{b} = -\vec{c} \quad \dots(i)$

$$\Rightarrow (\vec{a} + \vec{b}) \times \vec{b} = (-\vec{c}) \times \vec{b}$$

$$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{b}) = (-\vec{c}) \times \vec{b} \quad [\text{By distributive law}]$$

$$\Rightarrow (\vec{a} \times \vec{b}) + \vec{0} = (\vec{b} \times \vec{c})$$

$$[\because \vec{b} \times \vec{b} = \vec{0} \text{ and } (-\vec{c}) \times \vec{b} = \vec{b} \times \vec{c}]$$

$$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c} \quad \dots(ii)$$

Again from (i), $\vec{b} + \vec{c} = -\vec{a}$

$$\Rightarrow (\vec{b} + \vec{c}) \times \vec{c} = (-\vec{a}) \times \vec{c}$$

$$\Rightarrow (\vec{b} \times \vec{c}) + \vec{0} = \vec{c} \times \vec{a} \quad [\because \vec{c} \times \vec{c} = \vec{0} \text{ and } (-\vec{a}) \times \vec{c} = \vec{c} \times \vec{a}]$$

$$\Rightarrow \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \quad \dots(iii)$$

From (ii) and (iii), $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

15. Let $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

A vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$.

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$$= (4+4)\hat{i} + (4-12)\hat{j} + (-6-2)\hat{k} = 8(\hat{i} - \hat{j} - \hat{k})$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{64+64+64} = 8\sqrt{3}$$

$\therefore$ A unit vector perpendicular to $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{8(\hat{i} - \hat{j} - \hat{k})}{8\sqrt{3}} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

$$\text{Now, } a = |\vec{a}| = \sqrt{9+1+4} = \sqrt{14},$$

$$b = |\vec{b}| = \sqrt{4+4+16} = \sqrt{24}$$

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} = \frac{8\sqrt{3}}{\sqrt{14}\sqrt{24}} = \frac{2}{\sqrt{7}}$$

16. Let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$ be the required vector.

$$\text{Then, } x^2 + y^2 + z^2 = d^2 = 51 \quad \dots(i)$$

Let $\vec{d}$ makes equal angle θ with vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

$$\text{Then, } \vec{a} \cdot \vec{d} = \frac{1}{3}(x - 2y + 2z) \Rightarrow 3\sqrt{51} \cos \theta = x - 2y + 2z \quad \dots(ii)$$

$$\vec{b} \cdot \vec{d} = -\frac{1}{5}(4x + 3z) \Rightarrow 5\sqrt{51} \cos \theta = -4x - 3z \quad \dots(iii)$$

$$\text{and } \vec{c} \cdot \vec{d} = y \Rightarrow \sqrt{51} \cos \theta = y \quad \dots(iv)$$

Putting (iv) in (ii) and (iii), we get

$$x - 5y + 2z = 0 \quad \dots(v)$$

$$\text{and } 4x + 5y + 3z = 0 \quad \dots(vi)$$

Solving (v) and (vi), we get

$$\frac{x}{-15-10} = \frac{y}{8-3} = \frac{z}{5+20} \Rightarrow \frac{x}{-5} = \frac{y}{1} = \frac{z}{5} = \lambda \text{ (say)}$$

$$\therefore x = -5\lambda, y = \lambda, z = 5\lambda$$

Putting these values of x, y, z in (i), we get

$$51\lambda^2 = 51 \Rightarrow \lambda = \pm 1$$

Hence the required vector

$$\vec{d} = \mp(\hat{5i} - \hat{j} - 5\hat{k}) \text{ one of which is } -\hat{5i} + \hat{j} + 5\hat{k}$$

$$\begin{aligned} 17. \text{ L.H.S.} &= (1+|\vec{a}|^2)(1+|\vec{b}|^2) = (1+a^2)(1+b^2) \\ &= 1+a^2+b^2+a^2b^2 \quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= (1-\vec{a} \cdot \vec{b})^2 + |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|^2 \\ &= (1-ab\cos\theta)^2 + (\vec{a} + \vec{b} + (\vec{a} \times \vec{b})) \cdot (\vec{a} + \vec{b} + (\vec{a} \times \vec{b})), \\ &\quad \text{where } \theta \text{ is the angle between vectors } \vec{a} \text{ and } \vec{b} \\ &= 1-2ab\cos\theta + a^2b^2\cos^2\theta + \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot (\vec{a} \times \vec{b}) \\ &\quad + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{b}) \cdot \vec{a} + (\vec{a} \times \vec{b}) \cdot \vec{b} \\ &\quad + (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{b}) \\ &= 1-2ab\cos\theta + a^2b^2\cos^2\theta + a^2 + ab\cos\theta + 0 + \\ &\quad ab\cos\theta + b^2 + 0 + 0 + 0 + |\vec{a} \times \vec{b}|^2 \\ &= 1+a^2b^2\cos^2\theta + a^2 + b^2 + (ab\sin\theta)^2 \\ &= 1+a^2+b^2+a^2b^2(\cos^2\theta + \sin^2\theta) \\ &= 1+a^2+b^2+a^2b^2 \quad \dots(ii) \end{aligned}$$

From (i) and (ii), L.H.S. = R.H.S.

$$\begin{aligned} 18. \text{ Let } \lambda &= |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \\ \Rightarrow \lambda &= (|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}) + (|\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b} \cdot \vec{c}) \\ &\quad + (|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a}) \\ \Rightarrow \lambda &= \{2-2(\vec{a} \cdot \vec{b})\} + \{2-2(\vec{b} \cdot \vec{c})\} + \{2-2(\vec{c} \cdot \vec{a})\} \\ &\quad [\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1] \\ \Rightarrow \lambda &= 6 - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \\ \Rightarrow 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) &= 6 - \lambda \quad \dots(i) \end{aligned}$$

For any vector $\vec{r}$, we have $|\vec{r}|^2 \geq 0$.

$$\begin{aligned} \therefore |\vec{a} + \vec{b} + \vec{c}|^2 &\geq 0 \\ \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) &\geq 0 \\ \Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) &\geq 0 \quad [\because |\vec{a}|^2 = |\vec{b}|^2 = |\vec{c}|^2 = 1] \\ \Rightarrow 3 + 6 - \lambda &\geq 0 \quad \text{[From (i)]} \\ \Rightarrow 9 - \lambda &\geq 0 \Rightarrow \lambda \leq 9 \\ \Rightarrow |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 &\leq 9 \end{aligned}$$

19. Since it is given that

Projection of $\vec{b}$ along $\vec{a}$ = Projection of $\vec{c}$ along $\vec{a}$

$$\Rightarrow \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \Rightarrow \vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a} \Rightarrow \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \quad \dots(i)$$

Also, $\vec{b}$ and $\vec{c}$ are perpendicular to each other.

$$\Rightarrow \vec{b} \cdot \vec{c} = 0 \quad \dots(ii)$$

$$\begin{aligned} \text{Now, } |3\vec{a} - 2\vec{b} + 2\vec{c}|^2 &= 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 + \\ &\quad 2\{-6(\vec{a} \cdot \vec{b}) - 4(\vec{b} \cdot \vec{c}) + 6(\vec{a} \cdot \vec{c})\} \\ \Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}|^2 &= 9 \times 1^2 + 4 \times 2^2 + 4 \times 3^2 + \\ &\quad 2\{-6(\vec{a} \cdot \vec{c}) - 4 \times 0 + 6(\vec{a} \cdot \vec{c})\} \\ &\quad \text{[Using (i) and (ii)]} \end{aligned}$$

$$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}|^2 = 9 + 16 + 36 + 2 \times 0 = 61$$

$$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$$

20. It is given that $\vec{A}, \vec{B}, \vec{C}$ are non-coplanar vectors.

$$\therefore [\vec{A}, \vec{B}, \vec{C}] \neq 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$$\text{Now, } \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \quad \text{[Given]}$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

[Applying $C_2 \leftrightarrow C_3$ in 1st det. and taking a, b, c common from R_1, R_2, R_3 of 2nd det.]

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0 \quad \text{[Applying } C_2 \leftrightarrow C_1 \text{ in 1st determinant]}$$

$$\Rightarrow (1+abc) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc)[\vec{A}, \vec{B}, \vec{C}] = 0$$

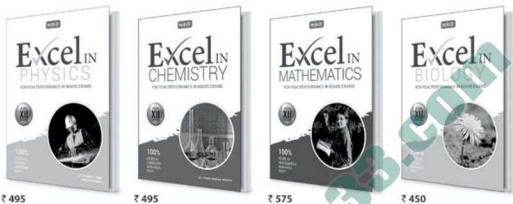
$$\Rightarrow 1+abc = 0$$

$$\Rightarrow abc = -1$$

$$[\because [\vec{A}, \vec{B}, \vec{C}] \neq 0]$$



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MPP-7

MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Definite Integration, Application of Integrals

Total Marks : 80

Time Taken : 60 Min.

Only One Option Correct Type

1. If $f(x) = \begin{cases} [3x] - 5 \frac{[x]}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ then $\int_{-3/2}^2 f(x) dx$,

where $[.]$ denotes the greatest integer function, is equal to

- (a) -7 (b) -6.5 (c) -6 (d) -5.5

2. The value of $\int_0^1 \frac{e^x(1+x)\sin^2(xe^x)dx}{\sin^2(xe^x) + \sin^2(e-xe^x)}$ is

- (a) $\frac{e}{4}$ (b) $\frac{e}{2}$ (c) $2e$ (d) $\frac{e}{8}$

3. Area of the ellipse represented by $3x^2 + 4xy + 3y^2 = 1$, is equal to

- (a) $\frac{\pi}{\sqrt{5}}$ sq. units (b) $\frac{\pi}{3\sqrt{5}}$ sq. units
(c) $\frac{\pi}{2\sqrt{5}}$ sq. units (d) $\frac{\pi}{4\sqrt{5}}$ sq. units

4. If $f'''(x) = k$ in $[0, a]$, then

$$\int_0^a f(x) dx - \left\{ xf(x) - \frac{x^2}{2!} f'(x) + \frac{x^3}{3!} f''(x) \right\}_0^a$$

- (a) $-ka^4/12$ (b) $ka^4/24$
(c) $-ka^4/24$ (d) None of these

5. If $\int_{-1}^{-4} f(x) dx = 4$ and $\int_2^{-4} (3-f(x)) dx = 7$, then the

value of $\int_{-2}^1 f(-x) dx$, is

- (a) 2 (b) -3
(c) 5 (d) None of these

6. If A_n is the area bounded by $y = x$ and $y = x^n$, $n \in N$, then $A_2 \cdot A_3 \dots A_n =$

- (a) $\frac{1}{n(n+1)}$ (b) $\frac{1}{2^n n(n+1)}$
(c) $\frac{1}{2^{n-1} n(n+1)}$ (d) $\frac{1}{2^{n-2} n(n+1)}$

One or More Than One Option(s) Correct Type

7. If $I_1 = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$, $I_2 = \int_0^{\pi} x \sin^4 x dx$, then

- $I_1 : I_2 =$
(a) 3 : 4 (b) 1 : 2 (c) 4 : 3 (d) 2 : 3

8. The integral $\int_0^{\pi} xf(\sin x) dx$ is equal to

- (a) $\frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$ (b) $\frac{\pi}{4} \int_0^{\pi} f(\sin x) dx$
(c) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x) dx$ (d) $\pi \int_0^{\pi/2} f(\cos x) dx$

9. For which of the following values of m , is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ is equal to $9/2$ sq. units?

- (a) -4 (b) -2 (c) 2 (d) 4

10. The area enclosed by the curves $x = a \sin^3 t$ and $y = a \cos^3 t$ is equal to

- (a) $12a^2 \int_0^{\pi/2} \cos^4 t \sin^2 t dt$
(b) $12a \int_0^{\pi/2} \cos^2 t \sin^4 t dt$

$$(c) \int_{-a}^a (a^{2/3} - x^{2/3})^{3/2} dx$$

$$(d) \int_0^a (a^{2/3} - x^{2/3})^{3/2} dx$$

11. The absolute value of $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$ is

- (a) less than 10^{-7} (b) more than 10^{-7}
(c) less than 10^{-6} (d) None of these

12. Area enclosed by parabola $ay = 3(a^2 - x^2)$ and x -axis is 64, then value of a is

- (a) 4 (b) 2 (c) -4 (d) -2

13. Consider the function

$$f(x) = \begin{cases} \int_0^x (4 + |t-2|) dt, & x > 3 \\ ax^2 + bx, & x \leq 3 \end{cases}$$

If $f(x)$ is differentiable at $x = 3$, then

- (a) $a+b = \frac{83}{18}$ (b) $a+b = \frac{85}{18}$
(c) $ab = \frac{7}{27}$ (d) $\frac{b}{a} = 84$

Comprehension Type

Consider $f(x) = \int_0^{\pi} \sin x \cdot f(t) dt + \int_0^{\pi/2} \cos xf'(t) dt + 2$

14. Range of $\frac{f(x)-2}{\pi}$ is

- (a) $[-\sqrt{2}, \sqrt{2}]$ (b) $[-\sqrt{5}, \sqrt{5}]$
(c) $[-\sqrt{3}, \sqrt{3}]$ (d) None of these

15. $\int_{\frac{\pi}{3}}^{\frac{13\pi}{3}} f(x) dx$ is equal to

- (a) 4π (b) 0 (c) 2 (d) 8π

Matrix Match Type

16. $\lim_{n \rightarrow \infty} f(n)$, where $f(n)$ is

Column-I	Column-II
P. $\left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2-1}} + \dots + \frac{1}{\sqrt{n^2-(n-1)^2}} \right)$	1. $\frac{\pi}{4} + \frac{1}{2} \log 2$
Q. $\left(\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \dots + \frac{1}{n} \right)$	2. 1/2
R. $\frac{1}{n} \left[\sin^2 \frac{\pi}{2n} + \sin^2 \frac{2\pi}{2n} + \dots + \sin^2 \frac{n\pi}{2n} \right]$	3. $\pi/6$
S. $\left(\frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-2^2}} + \dots + \frac{1}{\sqrt{4n^2-n^2}} \right)$	4. $\pi/2$

P	Q	R	S
(a) 4	1	2	3
(b) 2	1	3	4
(c) 1	2	3	4
(d) 4	2	1	3

Integer Answer Type

17. A biquadratic function $y = ax^4 + bx^3 + cx^2 + dx + e$ ($a \neq 0$) touches $y = px + q$ at $x = \alpha, \beta$ ($\alpha < \beta$). Area of the region bounded by these graphs is $\frac{a(\beta-\alpha)^5}{\lambda}$,

then $\frac{\lambda}{5} =$

18. If $I = \int_{-1}^1 \frac{e^{|x|}}{1+a^x} dx$, then the value of $\frac{9997}{(e-1)} I$ must be

19. If $\int_0^1 \frac{dx}{(1+x)(2+x)\sqrt{x(1-x)}} = \frac{\pi A}{\sqrt{6}(\sqrt{3}+1)(157)}$,

then A is equal to

20. If a' ($a > 0$) is the value of parameter for each of which the area of the figure bounded by the straight line $y = \frac{a^2 - ax}{1+a^4}$ and the parabola $y = \frac{x^2 + 2ax + 3a^2}{1+a^4}$ is the greatest, then the value of a^4 is



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No. of questions attempted
No. of questions correct
Marks scored in percentage

Check your score! If your score is

- > 90% EXCELLENT WORK ! You are well prepared to take the challenge of final exam.
90-75% GOOD WORK ! You can score good in the final exam.
74-60% SATISFACTORY ! You need to score more next time.
< 60% NOT SATISFACTORY ! Revise thoroughly and strengthen your concepts.



QUANTITATIVE APTITUDE TEST

Useful for Bank PO, Specialist Officers & Clerical Cadre,
BCA, MAT, CSAT, CDS and other such examinations

- A three digit number has digits a, b, c from the left to right with $a > c$. If the digits are reversed & the number thus formed is subtracted from the original number, the unit digit is 3. What are the other two digits from left to right?
(a) 6 & 9 (b) 9 & 6 (c) 6 & 3 (d) 9 & 3
- A person has five iron rods with lengths 16, 24, 48, 72, 104 cm each. He wants to convert into pieces of equal length from each of five rods. The least number of total pieces, if there is no wastage of material is
(a) 14 (b) 8 (c) 33 (d) 44
- In a university, one third boys and half of the girls participate in the camp. Out of the total participants of 300 students, 100 are boys then find the total number of students in the university.
(a) 600 (b) 800 (c) 500 (d) 700
- If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, then $y - x$ is equal to
(a) 3 (b) 4 (c) 7 (d) 2
- The square root of $\left(\frac{1\frac{1}{2}}{2}\right)^4 - \left(\frac{1\frac{1}{8}}{8}\right)^4$ is
(a) $1\frac{7}{8}$ (b) $\frac{7}{8}$ (c) $2\frac{7}{8}$ (d) $1\frac{7}{64}$
- The cube root of $135\sqrt{3} - 87\sqrt{6}$ equals
(a) $3\sqrt{2} + \sqrt{6}$ (b) $3\sqrt{3} - \sqrt{6}$
(c) $2\sqrt{3} - \sqrt{6}$ (d) None of these
- Mahavir purchase 1000 articles at the rate ₹ 5 each and sold 850 articles at the rate ₹ 7 each and rest articles at the rate ₹ 3.50 each. Find the average profit per article sold.
(a) ₹ 1.50 (b) ₹ 2.47 (c) ₹ 1.47 (d) ₹ 1.75
- The sum of three numbers is 264. If the first number be doubled the second and third number be one-third of the first, then the second number is
(a) 48 (b) 54 (c) 72 (d) 84
- 10 years before, the ratio of ages of A and B was 13 : 17. 17 years from now, the ratio of their ages will be 10 : 11. The present age of B (in years) is
(a) 23 (b) 27 (c) 40 (d) 44
- The average age of family of five members is 24 years. If the present age of youngest member is 8 years. What was the average age of the family at the time of birth of youngest member?
(a) 16 years (b) 18 years
(c) 20 years (d) 21 years
- The tank full of petrol in Arun's motor cycle lasts for 10 days. If he starts using 25% more everyday, in how many days will the tank full of petrol last?
(a) 6 (b) 7 (c) 8 (d) 5
- Rajni purchased a mobile phone and a refrigerator for ₹ 12000 and ₹ 10000 respectively, she sold the refrigerator at a loss of 12% and mobile phone at a profit of 8%. What is the overall loss/profit in whole transaction?
(a) loss of ₹ 280 (b) loss of ₹ 240
(c) profit of ₹ 2060 (d) loss of ₹ 640
- A merchant earn a profit of 20% by selling a basket containing 80 apples, which cost ₹ 240, but he gave $\frac{1}{4}$ of it to his friend at cost price and sells the remaining apple. In order to earn the same profit, at what price he sells each apple?
(a) ₹ 3.00 (b) ₹ 3.60 (c) ₹ 3.80 (d) ₹ 4.80
- Two vessels are full of milk with milk-water ratio 1 : 3 and 3 : 5 respectively. If both are mixed in the ratio 3 : 2, the ratio of milk and water in the new mixture is
(a) 4 : 15 (b) 3 : 7 (c) 3 : 10 (d) 6 : 7

15. Anil is an active and Vimal is a sleeping partner in a business. Anil invests ₹ 12000 and Vimal invests ₹ 20000. Anil received 10% profit for managing and the rest being divided in proportion to their capitals. Out of the total profit ₹ 9000, the money received by Anil is
(a) ₹ 4800 (b) ₹ 3937.50
(c) ₹ 4600 (d) ₹ 4500
16. A and B started a business with ₹ 20000 and ₹ 35000 respectively. They agreed to share the profit in the ratio of their capital. C joins the partnership with the condition that A, B and C will share profit equally and pays ₹ 220000 as premium for this, to be shared between A and B. This is to be divided between A and B in the ratio of
(a) 10 : 9 (b) 1 : 10 (c) 10 : 1 (d) 9 : 10
17. A daily wages worker appointed on a contract is paid ₹ 350 every day. He attends work and ₹ 125 is deducted from his salary as a fine every day remains absent. If in a month of 31 working days, he earned ₹ 8475, then for how many days he is absent?
(a) 5 (b) 6 (c) 7 (d) 8
18. 2000 soldiers in a fort had enough food for 20 days. But some soldiers were transferred to another fort and food lasted for 25 days. How many soldiers were transferred?
(a) 525 (b) 500 (c) 450 (d) 400
19. Nishtha can do a piece of work in 25 days and Tina can finish it in 20 days. They work together for 5 days then Nishtha quit herself. In how many days will Tina finish the remaining work?
(a) 20 (b) 18
(c) 15 (d) None of these
20. Through an inlet, a tank takes 8 hours to get filled up. Due to a leak in the bottom it takes 2 hours more to get it filled completely, if the tank is full, how much time will the leak take to empty it?
(a) 20 hours (b) 25 hours
(c) 40 hours (d) 30 hours
21. Two pipes P and Q can fill a cistern in 12 and 15 minutes respectively. If both are opened together and at the end of 3 minutes the tap P is closed. How much longer will the cistern take to fill?
(a) $8\frac{1}{4}$ minutes (b) $8\frac{1}{2}$ minutes
(c) $8\frac{3}{4}$ minutes (d) $9\frac{1}{4}$ minutes
22. A train of length 150 m takes 10 s to cross another train 100 m long coming from opposite direction. If speed of first train is 30 km/h. What is the speed of second train?
(a) 60 km/h (b) 55 km/h
(c) 50 km/h (d) 45 km/h
23. A motor boat takes 2 h to travel a distance of 9 km down the current and it takes 6 h to travel the same distance against the current. What is the speed of current (stream or water flow)?
(a) 3 km/h (b) 2 km/h
(c) 1.5 km/h (d) 2.5 km/h
24. In two types of stainless steel, the ratio of chromium and steel are in ratio 2 : 11 and 5 : 21. In what proportion should the two types be mixed, so that the ratio of chromium to steel in the mixture becomes 7 : 32?
(a) 2 : 3 (b) 3 : 4 (c) 1 : 2 (d) 1 : 3
25. How many kilograms of the tea powder costing ₹ 34 per kg be mixed with 33 kg of tea powder costing ₹ 42 per kg, such that the mixture when sold at ₹ 46 per kg gives a profit of 15%?
(a) 18 kg (b) 15 kg (c) 14 kg (d) 11 kg
26. An amount is invested in a bank at compound rate of interest. The total amount including interest after first year and third year is ₹ 1200 and ₹ 1587 respectively. What is the rate of interest?
(a) 10% (b) 12% (c) 15% (d) 20%
27. ABC is a triangle right angled at A, $AB = 6$ cm, $AC = 8$ cm. Semi circles are drawn (out side the triangle) on the sides AB, AC & BC as diameters which enclose the area x, y and z respectively. Then $z + y$ equals
(a) 17π cm² (b) $9\pi/2$ cm²
(c) 20.5π cm² (d) $25\pi/2$ cm²
28. A cylindrical box of radius 4 cm consisting of 6 solid spherical balls, each of radius same as the radius of cylindrical box. If the upper most ball touches upper cover of the box, then volume of the empty space in the box is
(a) 16π cm³ (b) 64π cm³
(c) 256π cm³ (d) None of these
29. The clock A and B began to strike 12 together. Clock A strikes its strokes in 33 seconds and the clock B strikes its strokes in 22 seconds. What is the difference of interval between the 5th stroke of clock A and the 7th stroke of the clock B?
(a) 0 sec (b) 2 secs (c) 3 secs (d) 4 secs

30. If $\sin x + \cos x = p$ & $\sin^3 x + \cos^3 x = q$, then what is $p^3 - 3p$ equals?

(a) 0 (b) $-2q$ (c) $2q$ (d) $4q$

SOLUTIONS

1. (a): Let original number = $100a + 10b + c$... (i)
After interchanging the digits number = $a + 10b + 100c$... (ii)

Now, subtracting (ii) from (i), we have

Required number = $99(a - c)$

As, unit digit in 99 ($a - c$) is 3, so $a - c = 7$

$\therefore$ 3 digit number = $99 \times 7 = 693$

2. (c) 3. (d) 4. (b) 5. (a)

6. (b): $135\sqrt{3} - 87\sqrt{6} = 3\sqrt{3}(45 - 29\sqrt{2})$

$$\therefore (135\sqrt{3} - 87\sqrt{6})^{1/3} = \sqrt{3}(45 - 29\sqrt{2})^{1/3}$$

Again, let $(45 - 29\sqrt{2})^{1/3} = x - \sqrt{y}$... (i)

$$\therefore (45 + 29\sqrt{2})^{1/3} = x + \sqrt{y} \quad \dots (ii)$$

Now multiplying (i) and (ii), we get

$$(2025 - 1682)^{1/3} = x^2 - y \Rightarrow x^2 - y = (343)^{1/3}$$

$$\Rightarrow y = x^2 - 7$$

Again cubing (i), we get

$$45 - 29\sqrt{2} = x^3 - 3x^2\sqrt{y} + 3xy - y\sqrt{y} \quad \dots (iii)$$

Equating rational parts of (iii)

$$x^3 + 3xy = 45 \Rightarrow 4x^3 - 21x - 45 = 0$$

$$\Rightarrow x = 3 \text{ (by using remainder theorem)}$$

$$\therefore y = 9 - 7 = 2$$

$$\text{Hence, } (135\sqrt{3} - 87\sqrt{6})^{1/3} = \sqrt{3}(3 - \sqrt{2})$$

7. (c)

8. (c): Let the second number be x .

Then first number is $2x$ and third number is $\frac{2x}{3}$.

$$\text{According to problem, } 2x + x + \frac{2x}{3} = 264$$

$$\Rightarrow 11x = 3 \times 264 \Rightarrow x = 72$$

9. (b): In the problem $x = 13k$, $y = 17k$

$\therefore$ Present age of A is $13k + 10$ and B is $17k + 10$

$\therefore$ According to problem, we have

$$\frac{13k + 10 + 17}{17k + 10 + 17} = \frac{10}{11} \Rightarrow 27k = 27 \Rightarrow k = 1$$

$\therefore$ Present age of B = $17k + 10 = 17(1) + 10 = 27$ years

10. (c)

11. (c): Let the quantity of petrol used everyday be x .

$\therefore$ Quantity of petrol used for 10 days = $10x$

= total petrol available

As the petrol used 25% more everyday

$$\therefore \text{Quantity of petrol used everyday} = x + \frac{x}{4} = \frac{5x}{4}$$

$\therefore$ Required number of days

$$= \frac{\text{Total petrol available}}{\text{Petrol used everyday}} = \frac{10x}{\frac{5x}{4}} = \frac{40}{5} = 8$$

12. (b)

13. (c)

14. (c): The ratio of milk and water in new mixture be x .
Now, according to problem, and using law of mixture we have

$$\frac{\frac{3}{8} - x}{x - \frac{1}{4}} = \frac{3}{2} \Rightarrow \frac{3 - 8x}{4x - 1} = \frac{3}{1} \Rightarrow x = \frac{3}{10}$$

$\therefore$ Required ratio is 3 : 10

15. (b): Anil's share for managing the business = 10% of ₹ 9000 = ₹ 900

The profit which is distributed = ₹ 8100

Now ratio of investment = 12000 : 20000 = 3 : 5

$$\therefore \text{Anil's share} = 8100 \times \frac{3}{8} = 3037.50$$

$\therefore$ Total amount received by Anil = ₹ 3937.50

16. (c) 17. (a) 18. (d) 19. (d)

20. (c): Let the leak takes x hours to empty the tank.

Now, part filled by inlet in 1 hour = $\frac{1}{8}$

Part filled by inlet and leak (outlet) together in 1

$$\text{hour} = \frac{1}{8 + 2} = \frac{1}{10}$$

$$\text{Now, according to problem } \frac{1}{x} = \frac{1}{8} - \frac{1}{10} \Rightarrow x = 40$$

21. (a)

22. (a): Speed of Ist train = 30 km/h = $\frac{25}{3}$ m/s

Total length of both the trains = 250 m

Let speed of IInd train be x m/s

Total time = 10 s

$$= \frac{\text{Total distance}}{\text{Speed of Ist train} + \text{Speed of IInd train}}$$

$$\Rightarrow 10 = \frac{250}{25/3 + x} \Rightarrow \frac{250 \times 3}{3x + 25} = 10 \Rightarrow x = \frac{50}{3}$$

$$\therefore \text{Speed of IInd train} = \frac{50}{3} \times \frac{18}{5} \text{ km/h} = 60 \text{ km/h}$$

23. (c) 24. (c) 25. (d) 26. (c)

27. (c) 28. (c) 29. (a) 30. (b)



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MATHS MUSING

SOLUTION SET-190

1. (a) : We know, $A + B + C = \pi$

$$\Rightarrow \frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2} \Rightarrow \cot\left(\frac{A}{2} + \frac{B}{2}\right) = \cot\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\Rightarrow \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2} = \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} \quad \dots(i)$$

$\tan \frac{A}{2}, \tan \frac{B}{2}, \tan \frac{C}{2}$ are in H.P.

$\Rightarrow \cot \frac{A}{2}, \cot \frac{B}{2}, \cot \frac{C}{2}$ are in A.P.

$$\therefore 2 \cot \frac{B}{2} = \cot\left(\frac{A}{2}\right) + \cot\left(\frac{C}{2}\right) \quad \dots(ii)$$

From (i) and (ii), $\cot \frac{A}{2} \cot \frac{C}{2} = 3 \quad \dots(iii)$

$$\Rightarrow \frac{\cot \frac{A}{2} + \cot \frac{C}{2}}{2} \geq \sqrt{\cot \frac{A}{2} \cot \frac{C}{2}} \Rightarrow \cot\left(\frac{B}{2}\right) \geq \sqrt{3}$$

2. (d) : $x = a$ must satisfy $x^2 - 3x + 1 = 0$

i.e., $a^2 - 3a + 1 = 0 \therefore a^2 = 3a - 1$

and $a^6 + 1 = (a^2 + 1)(a^4 + 1 - a^2)$

$$\text{Now, } \frac{a^3}{a^6 + 1} = \frac{a^3}{3a(9a^2 + 1 - 6a + 1 - a^2)} = \frac{a^2}{18a^2} = \frac{1}{18}$$

3. (a) : $(x-1)(x^2-2)(x^3-3) \dots (x^n-n)$

Highest power of $x = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} = \alpha$ (say)

For the coefficient of $x^{\alpha-7}$ we should leave

$(x^7-7), (x-1)(x^6-6), (x^2-2)(x^5-5),$

$(x^3-3)(x^4-4), (x-1)(x^2-2)(x^4-4)$

If we leave (x^7-7) , coefficient = -7

If we leave $(x-1)(x^6-6)$ coefficient = 6

If we leave $(x^2-2)(x^5-5)$ coefficient = 10

If we leave $(x^3-3)(x^4-4)$ coefficient = 12

If we leave $(x-1)(x^2-2)(x^4-4)$ coefficient = -8

$\therefore$ Required coefficient = $12 + 10 + 6 - 7 - 8 = 13$

$$4. (a) : \text{Let } \Delta = \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \end{vmatrix}^2$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, then

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \beta-1 \\ 1 & \alpha^2-1 & \beta^2-1 \end{vmatrix} = (\alpha-1)^2(\beta-1)^2(\beta-\alpha)^2 \Rightarrow k = 1$$

5. (d) : $\because 0 < x < 1 \Rightarrow \frac{1}{2}x^2 < x^2 < x \Rightarrow -x^2 > -x$

$$\Rightarrow e^{-x^2} > e^{-x} \Rightarrow e^{-x^2} \cos^2 x > e^{-x} \cos^2 x$$

$$\Rightarrow \int_0^1 e^{-x^2} \cos^2 x dx > \int_0^1 e^{-x} \cos^2 x dx \Rightarrow I_2 > I_1 \quad \dots(i)$$

Now, $\cos^2 x \leq 1 \Rightarrow e^{-x^2} \cos^2 x < e^{-x^2} < e^{-(1/2)x^2}$

$$\Rightarrow \int_0^1 e^{-x^2} \cos^2 x dx < \int_0^1 e^{-x^2} dx < \int_0^1 e^{-(1/2)x^2} dx$$

$$\Rightarrow I_2 < I_3 < I_4 \quad \dots(ii)$$

From (i) and (ii), we get $I_1 < I_2 < I_3 < I_4$

6. (a,b,c) : $\because A = \begin{pmatrix} 5 & -3 \\ 111 & 336 \end{pmatrix} \therefore |A| = 1680 + 333 = 2013$

$$\therefore \det(-3A^{2013} + A^{2014}) = |A^{2013}(-3I + A)|$$

$$= |A^{2013}| \cdot |-3I + A| = 2013^{2013} \cdot 3^2(1 + 10 + 10^2)$$

$$\Rightarrow \alpha = 2013, \beta = 3, \gamma = 10$$

7. (c) : We have, $\frac{\cos^4 x}{a} + \frac{\sin^4 x}{b} = \frac{1}{a+b} = \frac{\cos^2 x + \sin^2 x}{a+b}$

$$\Rightarrow \cos^2 x \left(\frac{\cos^2 x}{a} - \frac{1}{a+b} \right) = \sin^2 x \left(\frac{1}{a+b} - \frac{\sin^2 x}{b} \right)$$

$$\Rightarrow \frac{\cos^2 x}{a} = \frac{\sin^2 x}{b}$$

8. (b)

9. (4) : Let $f(x) = p(x) - 10x \therefore f(1) = f(2) = f(3) = 0$

$\therefore p(x) = (x-1)(x-2)(x-3)(x-\lambda) + 10x$

$p(12) = 11 \times 10 \times 9(12-\lambda) + 120$

$p(-8) = (-9) \times (-10) \times (-11)(-8-\lambda) - 80$

$\therefore p(12) + p(-8) = 40 \times 496$

10. (b) : (P) $(\vec{a} + \vec{b} + \vec{c}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})]$

$= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{a} \times \vec{c}) + \vec{c} \cdot (\vec{b} \times \vec{a})$

$= [\vec{a} \vec{b} \vec{c}] + [\vec{b} \vec{a} \vec{c}] + [\vec{c} \vec{b} \vec{a}] = -3[\vec{a} \vec{b} \vec{c}]$

(Q) Taking $\vec{b} = \hat{i}, \vec{c} = \hat{j} \therefore \vec{b} \times \vec{c} = \hat{i} \times \hat{j} = \hat{k} \Rightarrow |\vec{b} \times \vec{c}| = 1$

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \hat{i} = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{i} = a_1, \vec{a} \cdot \vec{c} = \vec{a} \cdot \hat{j} = a_2$

$$\therefore (\vec{a} \cdot \vec{b})\vec{b} + (\vec{a} \cdot \vec{c})\vec{c} + \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|}(\vec{b} \times \vec{c}) = a_1 \hat{i} + a_2 \hat{j} + \frac{(\vec{a} \cdot \hat{k})\hat{k}}{|\hat{k}|^2}$$

$$= a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} = \vec{a}$$

(R) : $[\vec{a} + \vec{b} + \vec{c}, \vec{a} - \vec{c}, \vec{a} - \vec{b}] = (\vec{a} + \vec{b} + \vec{c}) \times (\vec{a} - \vec{c}) \cdot (\vec{a} - \vec{b})$

$= (-\vec{a} \times \vec{c} - \vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{b} \times \vec{a}) \cdot (\vec{a} - \vec{b})$

$$= -[\vec{b} \vec{c} \vec{a}] + [\vec{a} \vec{c} \vec{b}] - [\vec{c} \vec{a} \vec{b}] = -3[\vec{a} \vec{b} \vec{c}]$$



YOU ASK WE ANSWER

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1. $\bar{b}z + b\bar{z} = c$ is the equation of a straight line. If z_1 and z_2 be mirror image of each other in this line, then prove that $\bar{b}z_2 + b\bar{z}_1 = c$. (Abhay, Delhi)

Ans. z_1 and z_2 are mirror images of each other in the line $\bar{b}z + b\bar{z} = c$... (i)

Putting $z = x + iy$ in equation (i), we get

$$\bar{b}(x + iy) + b(x - iy) - c = 0 \Rightarrow (\bar{b} + b)x + i(\bar{b} - b)y - c = 0$$

whose slope = $\frac{(b + \bar{b})}{i(b - \bar{b})}$.

Also, line joining z_1, z_2 must be perpendicular to the above line. ... (ii)

The mid point of z_1, z_2 must lie on the line (i).

$$\therefore \bar{b}\left(\frac{z_1 + z_2}{2}\right) + b\left(\frac{\bar{z}_1 + \bar{z}_2}{2}\right) = c$$

$$\Rightarrow (\bar{b}z_1 + b\bar{z}_2) + (\bar{b}z_2 + b\bar{z}_1) = 2c \quad \dots (iii)$$

$$\text{Equation of the line joining } z_1, z_2 \text{ is } \begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

i.e., $z\bar{a} - \bar{z}a + d = 0$, where $a = z_1 - z_2$ and $d = z_1\bar{z}_2 - \bar{z}_1z_2$

Putting $z = x + iy$, in the above equation, we get

$$\bar{a}(x + iy) - a(x - iy) + d = 0 \Rightarrow (\bar{a} - a)x + i(\bar{a} + a)y + d = 0$$

whose slope = $\frac{(a - \bar{a})}{i(a + \bar{a})}$

Satisfying (ii), we have $\frac{(b + \bar{b})}{i(b - \bar{b})} \times \frac{(a - \bar{a})}{i(a + \bar{a})} = -1$

$$\Rightarrow (b + \bar{b})(a - \bar{a}) = (b - \bar{b})(a + \bar{a}) \Rightarrow \bar{b}a = b\bar{a}$$

$$\Rightarrow \bar{b}(z_1 - z_2) = b(\bar{z}_1 - \bar{z}_2) \Rightarrow \bar{b}z_1 + b\bar{z}_2 = \bar{b}z_2 + b\bar{z}_1 \quad \dots (iv)$$

From (iii) and (iv), we have $\bar{b}z_2 + b\bar{z}_1 = c$

2. Evaluate: $\lim_{x \rightarrow 0} \frac{[f(x)] + x^2}{[f(x)]}$

where $f(x) = \frac{\tan x}{x}$ and $[\cdot], \{\cdot\}$ denote integral and fractional parts respectively. (Suman, M.P.)

Ans. We have $L = \lim_{x \rightarrow 0} \frac{([f(x)] + x^2)^{\frac{1}{[f(x)]}}}{[f(x)]}$

$$= \lim_{x \rightarrow 0} \left[\frac{1 + [f(x)] + x^2 - 1}{[f(x)] + x^2 - 1} \right]^{\frac{1}{[f(x)]}}$$

Now, $\lim_{x \rightarrow 0} \left[\frac{\tan x}{x} \right] + x^2 - 1 = 1 + 0 - 1 = 0$ and

$$\lim_{x \rightarrow 0} \frac{[f(x)] + x^2 - 1}{[f(x)]} = \lim_{x \rightarrow 0} \frac{\left[1 + \frac{x^2}{3} + \frac{2x^4}{15} + \dots \right] + x^2 - 1}{\left[1 + \frac{x^2}{3} + \frac{2x^4}{15} + \dots \right]}$$

$$= \lim_{x \rightarrow 0} \frac{1 + x^2 - 1}{\frac{x^2}{3} + \frac{2x^4}{15} + \dots} = \lim_{x \rightarrow 0} \frac{1}{\frac{1}{3} + \frac{2x^2}{15} + \dots} = 3$$

$$\therefore L = \lim_{\delta \rightarrow 0} \left[(1 + \delta)^{\frac{1}{\delta}} \right]^3 = e^3$$

3. Show that

$$\int_0^{\pi} x^3 \ln(\sin x) dx = \frac{3\pi}{2} \int_0^{\pi} x^2 \ln(\sqrt{2} \sin x) dx$$

(Anuradha, W.B.)

Ans. We have $I = \int_0^{\pi} x^3 \ln(\sin x) dx$... (i)

$$\Rightarrow I = \int_0^{\pi} (\pi - x)^3 \ln \{ \sin(\pi - x) \} dx$$

$$\Rightarrow I = \int_0^{\pi} (\pi^3 - 3\pi^2 x + 3\pi x^2 - x^3) \ln(\sin x) dx \quad \dots (ii)$$

Adding (i) and (ii), we have

$$2I = \pi^3 \int_0^{\pi} \ln(\sin x) dx - 3\pi^2 \int_0^{\pi} x \ln(\sin x) dx + 3\pi \int_0^{\pi} x^2 \ln(\sin x) dx$$

$$= \pi^3(-\pi \ln 2) - 3\pi^2 I_1 + 3\pi \int_0^{\pi} x^2 \ln(\sin x) dx \quad \dots (iii)$$

Now, $I_1 = \int_0^{\pi} x \ln(\sin x) dx = \int_0^{\pi} (\pi - x) \ln(\sin x) dx$

Adding the integrals, we have

$$I_1 = \frac{\pi}{2} \int_0^{\pi} \ln(\sin x) dx = \frac{-\pi^2}{2} \ln 2$$

Hence, from (iii) we have

$$2I = -\pi^4 \ln 2 - 3\pi^2 \left(\frac{-\pi^2}{2} \ln 2 \right) + 3\pi \int_0^{\pi} x^2 \ln(\sin x) dx$$

i.e., $I = \frac{\pi^4}{4} \ln 2 + \frac{3\pi}{2} \int_0^{\pi} x^2 \ln(\sin x) dx$

$$= \frac{3\pi}{2} \left[\frac{\pi^3}{3} \ln \sqrt{2} + \int_0^{\pi} x^2 \ln(\sin x) dx \right]$$

$$= \frac{3\pi}{2} \left[\int_0^{\pi} x^2 \ln \sqrt{2} dx + \int_0^{\pi} x^2 \ln(\sin x) dx \right]$$

$$= \frac{3\pi}{2} \int_0^{\pi} x^2 \ln(\sqrt{2} \sin x) dx$$



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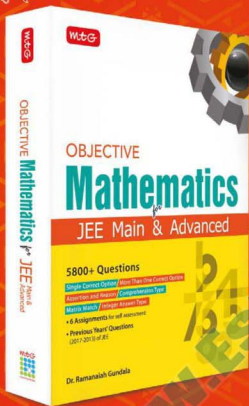
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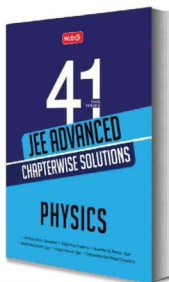
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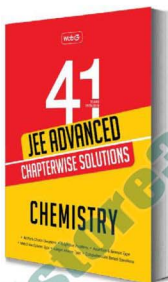
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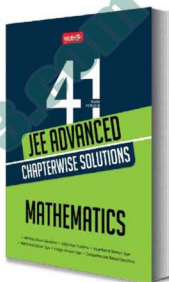
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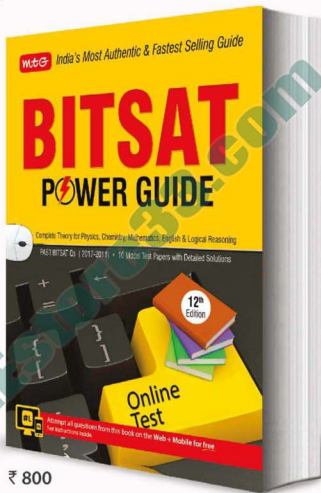
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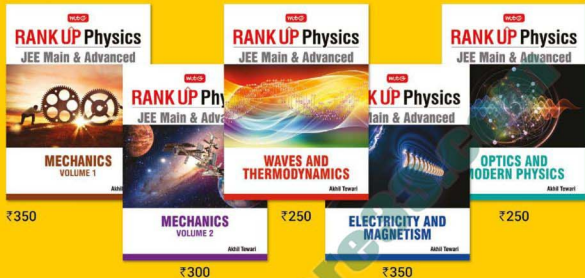
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